F2210701

MATH 1070 FINAL EXAM – Fall 2022

SHOW ALL WORK. DO NOT USE A CALCULATOR.

Each problem is worth 10 points.

1) Find the domain of the following function: $f(x) = \frac{3+x}{4x^2-x}$

2) Solve: $|2x - 1| + 7 \leq 14$

3) Write an equation for a function that has the shape of $y = \sqrt{x}$ but is reflected across the x-axis and shifted up 2 units.

4) Write a slope-intercept equation for a line passing through the point (2, -3) that is parallel to $x + 2y = 3$.

5) Given the function $f(x) = \begin{cases} -2x^2 & \text{for } x \leq -1 \\ 4 & \text{for } -1 < x \leq 3 \\ |1 - x| & \text{for } x > 3 \end{cases}$, find the following:

   a. $f(-2)$

   b. $f(4)$

6) The sum of two numbers is 42. Five times one number minus the other is also 42. Find the two numbers.

7) Given $f(x) = -\frac{1}{3}x$ and $g(x) = -x^2 + 4x + 3$, find:
   a) $(gf)(3)$

   b) $(f \circ g)(-1)$

   c) $(g \circ f)(x)$

8) Construct and simplify the difference quotient for $f(x) = 3x^2 - x$.

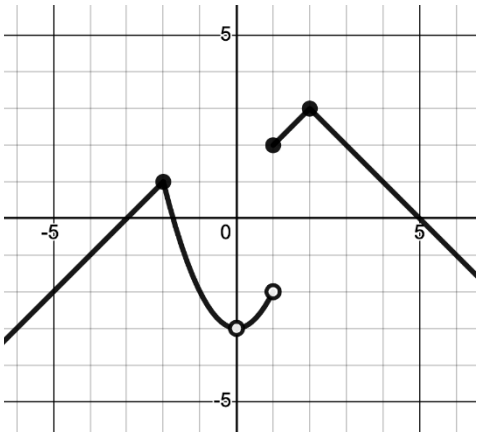
9) For the given graph of the function $f(x)$, find:

   a) The domain of $f(x)$

   b) The range of $f(x)$

   c) The intervals where $f(x)$ is decreasing

   d) $f(1)$



10) Solve: $\frac{x}{x-2} + \frac{6}{x^2-2x} = \frac{5}{x-2}$

11) Solve: $x = \sqrt{x+7} + 5$

12) The length of a rectangle is 2 cm longer than its width. If the width of the rectangle increases 3 cm while its length is reduced 2 cm, then the area of the new rectangle is 70 cm^2. Find the dimensions of the original rectangle.

13) Find:

a) $\log_4 \frac{1}{64}$ b) $\log_2 32$ c) $\log_{81} 3$

14) Solve $\log_6(x+5) = 2 - \log_6(x-4)$.

15) Solve, writing any non-real solutions in the form $a + bi$: $x^2 - 2x + 5 = 0$

16) Solve: $(x^2 - 3x)^2 - 14(x^2 - 3x) + 40 = 0$

17) Suppose you have clay with which to make a sculpture shaped as a rectangular prism. You want the height and width each to be 5 inches less than the length.

a) Write a polynomial that would find the volume of the prism as a function of *x*.

b) What should the dimensions of the prism be if you have 250 cubic inches of clay and want to use all of your clay?

18) Given: $g(x) = -2x^2 - 12x$

a. Find the vertex by completing the square.

b. Graph the function, labeling the vertex and all x- and y-intercepts.

19) Solve $\frac{x^2-3}{x-2} < 2$

20) A conic section is given by the equation $3y^2 - 75 = 3x^2$.

a. Identify the conic section.

b. Sketch the graph of the conic section. Plot and label all relevant points.