18002020091

MAT 1800 FINAL EXAM

Instructions:

- Please read the directions to each problem carefully.
- Each problem is worth 12 points except where indicated.
- Solutions should be written clearly and concisely on blank sheets of paper. **All work must be shown to receive full credit.** ***Answers without correct supporting work will receive minimal credit.***
- **No outside assistance of any kind is allowed.** This includes using the internet to find answers, using your notes, having another person look at your work before submission, looking at another person's work before submission, and/or sharing information in any way while completing the assessment.
- Calculators are not permitted.
- Webcams are required to be on for the duration of the exam.
- You will have 120 minutes to complete the Final Exam and an additional 15 minutes to upload your work on Canvas.

1. (8 pts each) Let $f(x) = 1 - 7x^2$ and $g(x) = \frac{x}{2x+1}$. Find and simplify each of the following.

(a) $(g \circ f)(x)$ (b) $g^{-1}(x)$

2. Sketch a graph of the function $g(x) = \begin{cases} |x-1| & if \quad x \le 2 \\ -(x-3)^2 & if \quad 2 < x < 4. \\ -2 & if \quad x \ge 4 \end{cases}$

3. Find the domain of the function $f(x) = \frac{\log_3(5+4x-x^2)}{2x-3}$. State your answer in **interval notation**.

4. Find the average rate of change of the function $g(x) = \frac{3}{x+1}$ from $x = a$ to $x = a + h$ and **simplify your answer** so that no single factor of h is left in the denominator.

5. Consider the polynomial function $p(x) = x^4 - 2x^3 + 3x^2 + 6x - 18$. Given that $x^2 - 3$ is a factor of $p(x)$, find all the zeros of the polynomial.

6. Let $f(x) = \frac{4(x-2)^2}{x(x+3)}$.

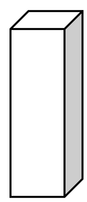
(a) Find all intercepts and asymptotes for $f(x)$.

(b) Sketch the graph of $f(x)$.

7. Solve the logarithmic equation: $\log(x-1) + \log(5) = 2 - \log(x)$.

8. Graph $f(x) = 4^{-x+2} - 1$ using transformations. Label all asymptotes and intercepts.

9. A cardboard box has a square base and a square top with each edge of the squares measuring *x* inches. The total length of all 12 edges of the box is 112 inches. Express the volume *V* of the box as a function of *x*.



10. (8 pts each) Find the exact value of each expression.

(a) $\log_6\left(7\sqrt{6}\right) + \log_6\left(\frac{1}{7}\right)$ (b) $4^{\log_4(24) - 3\log_4(2)}$

11. The population of a certain rare animal species increases exponentially according to the function $P(t) = P_0 e^{rt}$, where P_0 is the initial population, t is time measured in years, and r is a constant. If the population increases from 20 to 80 in 4 years, what will the population be in another 2 years? **Simplify** your answer as much as possible.

12. Find the exact value of each trigonometric function at the given real number if it exists.

(a) $\sin\left(-\frac{17\pi}{6}\right)$ (b) $\tan^{-1}(\cos(\pi))$

13. Given that $\csc\theta = 3$ and $\cos\theta < 0$, find the exact value of $\cos\left(\frac{4\pi}{3} - \theta\right)$.

14. State the amplitude and period length for the function $f(x) = 2\sin\left(\frac{1}{3}\pi x\right) - 1$. Graph this function over one complete period, labeling all high points and low points.

15. Verify that the trigonometric equation is an identity.

$$\sin x = (1 - \cos x)(\csc x + \cot x)$$

16. Find all primary solutions $(0 \leq x < 2\pi)$ of the equation: $\cos(2x) + \sin(x) = 1$.