18002022

MAT 1800 FINAL EXAM

Read the directions to each problem carefully. **ALL WORK MUST BE SHOWN IN THE PROVIDED BLUE BOOK.** Only minimal credit will be awarded for answers without supporting work. Each problem is worth 12 points except where indicated. **NO CALCULATORS ALLOWED.**

1. Sketch a graph of the function $f(x) = \begin{cases} -x^2 - 2x + 3 & \text{if } x < 0 \\ 4 & \text{if } 0 \le x < 3 \\ x + 1 & \text{if } x \ge 3 \end{cases}$

2. (8 points each) Let $m(x) = \sqrt{x+3}$ and $p(x) = x^2 - 5$

   a) Find and simplify $\frac{(p \circ m)(6)}{(p-m)(1)}$.

   b) Find $m^{-1}(2)$.

3. The hypotenuse of a right triangle is 3 inches less than twice the base of the triangle. Express the area of the triangle as a function of the base of the triangle.

4. Find the domain of the function $f(x) = \frac{\log_2(x^2+3x-4)}{x-5}$.

5. Given that $2x + 1$ is a factor of the polynomial, find all roots of the equation $2x^3 - 7x^2 + 6x + 5 = 0$. Express any non-real roots in the form $a+bi$.

6. Find the average rate of change of the function $n(x) = \frac{1}{x-3}$ from $x = 5$ to $x = 5 + h$. Simplify your answer completely.

7. Graph the function $g(x) = \frac{x+1}{(x-4)(x+3)^2}$, labeling all intercepts and asymptotes.

8. A local coffee shop wants to produce coffee cups. The shop has determined that when x coffee cups are made, the cost per coffee cup is determined by $C(x) = \frac{1}{2}x^2 - 8x + 68$.
   a) What is the minimum cost?
   b) How many coffee cups should be produced to yield the minimum cost?

9. Simplify each expression completely.

a) (6 points) $\log_4(\sqrt{8})$

b) (10 points) $49^{\log_7(3)+2\log_7(2)}$

10. A bacteria culture decays exponentially according to the function $Q(t) = Q_0 e^{rt}$. If the culture decays from 140 grams to 20 grams in 5 hours, find the time it takes for the population to decrease to half its initial size. Simplify your answer completely.

11. Graph $h(x) = -\log_2(x+4) + 3$. Label all intercepts and asymptotes.

12. (6 points each) Evaluate each of the following.

a) $\sec\left(\frac{-17\pi}{6}\right)$

b) $\sin^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right)$

13. Graph one complete period of the function $g(x) = 5\sin\left(\frac{1}{3}x\right) + 2$, labeling the highest and lowest points.

14. Given that $\cot(\theta) = -3$ and $\cos(\theta) < 0$, find $\sin\left(\theta - \frac{5\pi}{3}\right)$. Simplify your answer completely.

15. Find all primary solutions ($0 \le \theta < 2\pi$) of the trigonometric equation $2\cos^2(\theta) = 9\cos(\theta) + 5$

16. Verify the identity: $\frac{\sin(2x)}{1+\cos(2x)} = \tan(x)$