FINAL EXAMINATION, MAT 2010
December 16, 2022

INSTRUCTIONS
Write your solutions in a blue book. To receive full credit you must show *all* work. You are allowed to use an *approved* graphing calculator unless otherwise indicated. Simplify your answers when possible, but use the precise value rather than an approximation when you have a choice. (Example: If the actual answer is π, then write π, not 3.14.) The 12 problems are worth a total of 150 points. The time limit is 2 hours [120 minutes].

Cell phones are strictly prohibited!

1. (10 points) Use the **definition** of the derivative to differentiate the function

$$f(x) = \sqrt{5x-4}$$

(No credit will be awarded for calculating the derivative *without* using the definition of the derivative.)

2. (7 points each) Find the exact value of each of the following limits. Write "∞," "$-\infty$," or "does not exist" if appropriate. It is particularly important to show your work on this problem. Numerical approximations do not constitute an acceptable solution.

(a) $\displaystyle\lim_{x\to 2} \frac{\dfrac{1}{x+4} - \dfrac{1}{3x}}{x-2}$

(b) $\displaystyle\lim_{x\to 5^-} \frac{|x-5|}{(x-5)^2}$

(c) $\displaystyle\lim_{x\to -\infty} \frac{x^2}{e^{1-x}}$

3. (7 points each) Differentiate the following functions.

(a) $f(x) = \sqrt[3]{x^2} \cdot \Big(x+3\Big)^5$

(b) $h(x) = \ln\Big(\arctan(3x)\Big)$

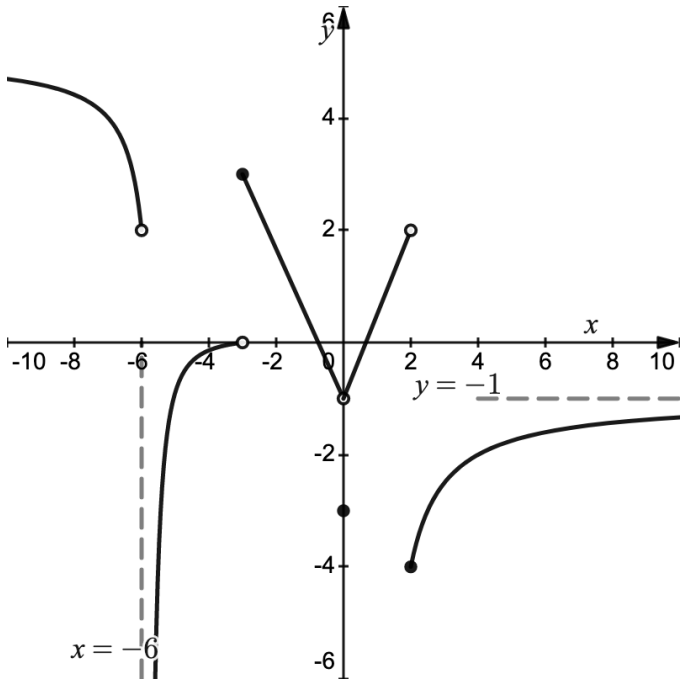
4. Evaluate.

(a) (7 points) $\int \left(\sqrt{2}\sec x \tan x + \sec^2 x - \frac{6}{\sqrt{1-x^2}}\right) dx$

(b) (8 points) $\int_1^e \left(x - \frac{1}{x}\right) dx$

(Give an exact answer in (b). Do not convert to decimals.)

5. (10 points) The graph of a function $f(x)$ is given below.



Find each of the following. [Use ∞, $-\infty$, or "does not exist" where appropriate.]

(a) $\lim_{x \to 2} f(x)$

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to -3^-} f(x)$

(d) $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

(e) $\lim_{x \to 0} f(x)$

6. (10 points) For the curve
$$x^2 + 2xy + 4y^2 = 13$$
   (a) Find $\dfrac{dy}{dx}$.
   (b) Find the equation of the tangent line to the curve at the point $(-1, 2)$. Write your answer in slope-intercept form.

7. (10 points) Sketch the graph of a function $g(x)$ that has **all** of the following properties:
   - g is defined for all real numbers except for $x = 0$.
   - g is continuous everywhere except at $x = -2$, 0, and 2.
   - g has a removable discontinuity at $x = -2$.
   - g has an infinite discontinuity at $x = 0$.
   - g has a jump discontinuity at $x = 2$.
   - $\lim\limits_{x \to -\infty} g(x) = \infty$
   - $\lim\limits_{x \to \infty} g(x) = 0$
   - $g(x)$ is continuous but not differetiable at $x = 4$

8. (10 points) A particle moves horizontally in a straight line with position function given by
$$s(t) = \sin^2 t + \cos t \qquad 0 \le t \le 2\pi,$$
   where, t is measured in minutes (min) and s is measured in feet (ft). When is the particle moving forward (moving in the positive direction)?

9. (10 points) You are given that
$$f''(x) = 3\sqrt{x} + 2, \qquad f(0) = 7, \qquad \text{and} \qquad f'(0) = 3.$$
   Find $f(x)$.

10. (10 points) Use the differential or a linear approximation to estimate $\sqrt{14.9}$. Give answer correct to two decimal places.

11. (10 points) The concentration of a drug in the blood stream, t hours after injection into the muscle tissue, is given by $C(t) = \dfrac{2t}{16 + t^2}$. When will the concentration have a maximum value?

12. (20 points) Given the following information for a function $f(x)$

- $f(x)$ is defined and continuous for all real numbers except at $x = -3$ and $x = 3$
- $x = -3$ and $x = 3$ are vertical asymptotes of $f(x)$
- $\lim_{x \to \infty} f(x) = \infty$
- $\lim_{x \to -\infty} f(x) = \infty$
- $f'(x) = \dfrac{2x}{x^2 - 9}$
- $f''(x) = -\dfrac{2(x^2 + 9)}{(x^2 - 9)^2}$
- $f(0) \approx 2.2$

Find

(a) All intervals on which $f(x)$ is increasing or decreasing.

(b) x-value(s) of all local (relative) extrema.

(c) All intervals where $f(x)$ is concave up or concave down.

(d) x-value(s) of all inflection points.

(e) Sketch the graph of $f(x)$. Label all asymptotes, local extrema, and inflection points.