Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, January 5, 2024
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number (*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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Problem 1 (10 points)

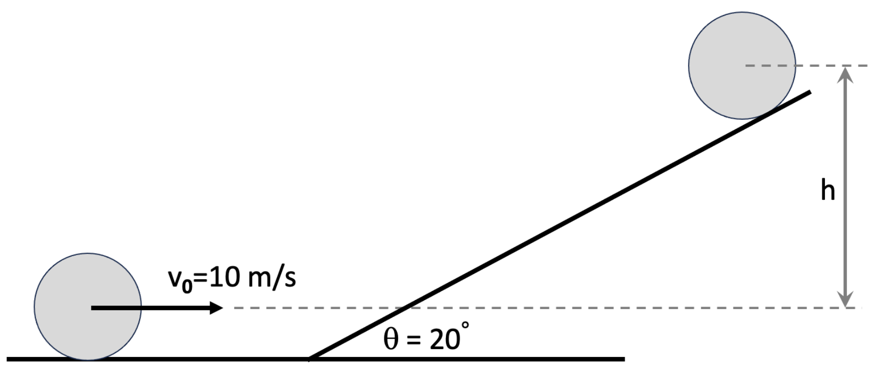
A solid sphere of mass, M, and radius, R, rolls without slipping over a horizontal surface at a speed of $v_0 = 10$ m/s. It encounters an incline at an angle $\theta = 20$ degrees. The incline also has a rough surface so the also ball rolls up the incline without slipping. The entrance to the ramp is smoothed so that at one instant the ball is rolling horizontally, and the next instant it is rolling up the ramp, without any bouncing at the transition.

a) (4 Points) Show that the moment of inertia, I, of the solid sphere is given by,

$$I = \frac{2}{5} M R^2. \tag{1}$$

b) (5 Points) Derive an expression for the height, h, (see figure) that the ball reaches (as it slows to a stop due to standard gravity after rolling up the incline) versus the initial velocity, v_0, and other constants.

c) (1 Points) Evaluate this expression for $v_0 = 10$ m/s and $\theta = 20$ degrees to determine h in meters.



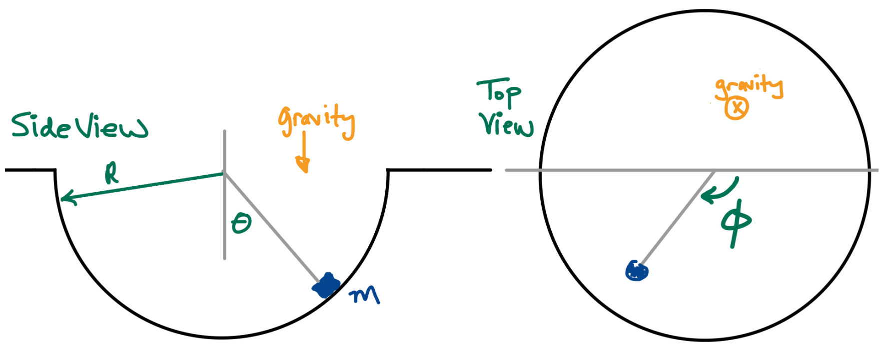
Problem 2 (10 points)

A particle of mass m is free to slide on a frictionless hemispherical bowl, such that its coordinates at any time are given by (R, θ, ϕ), where R is the constant radius of the bowl. The bowl opens upward" (see figure), and standard gravity points downward.

a) (4 Points) Write the Lagrangian for this particle.

b) (3 Points) Use the Euler-Lagrange equation to derive the Equation of Motion (EoM) for the θ coordinate, and the EoM for the ϕ coordinate.

c) (3 Points) Show that, for motion through the lowest point (*i.e.* ϕ=constant), the EoM for the θ coordinate takes a familiar form. Can you think of another mechanical system that has this same EoM?



Problem 3 (10 points)

The Lennard-Jones potential provides a reasonable description of many electrically neutral diatomic molecules. It describes the potential energy (in Joules) versus the distance x between the two particles in the molecule as,

$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right], \tag{2}$$

where ϵ and σ are positive constants (in Joules and meters, respectively).

a) (4 Points) What is the separation of the two particles at equilibrium, x_{eq}?

a) (3 Points) What is the potential energy, U, at the equilibrium separation x_{eq}?

b) (3 Points) What is the frequency, ω, of small oscillations about the equilibrium point?

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PART II

Friday, January 5, 2024
13:30 — 15:30

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Problem 4 (10 points)

Consider a leaky" spherical capacitor, namely a sphere with radius a surrounded by a concentric larger sphere with radius b. The region between the spheres is filled with a uniform medium with conductivity σ and dielectric permittivity ϵ. After connecting to a battery for some time, there is a charge $+Q$ on the outer shell and a charge $-Q$ on the inner shell at $t = 0$. Both spheres are conducting.

a) (4 points) Find the capacity of the configuration.
b) (4 points) Find the resistance of the configuration.
c) (2 points) Give the time τ over which those charges decrease by a factor $1/e$.

Problem 5 (10 points)

A static electric field is described by:

$$\mathbf{E} = \frac{V_0}{R} \exp\left(-r/R\right)\hat{\mathbf{r}}.$$

a) (2 points) Determine the charge density $\rho(\mathbf{r})$.
b) (2 points) Determine the total charge Q.
c) (2 points) Determine the potential $V(\mathbf{r})$.
d) (2 points) Determine the energy of the configuration.
e) (2 points) A small test charge $+q$ is released at $r = R \ln 2$. Find its kinetic energy at infinity .

Problem 6 (10 points)

A plane wave propagates in vacuum and is described by the equation

$$\mathbf{E}(\mathbf{r}, t) = \frac{V_0}{a} \cos\left(3x/a - 4y/a - \omega t\right)\hat{\mathbf{z}}.$$

In the following, give your answers as a function of V_0, a, and electromagnetic constants.

a) (2 points) Find ω and the period of the wave .
b) (2 points) Find the wavelength of the wave.
c) (3 points) Derive an equation for $\mathbf{B}$.
d) (3 points) Derive an equation for $\nabla \times \mathbf{E}$.

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PART III

Monday, January 8, 2024
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

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Problem 7 (10 points)

The system of N quantum oscillators (of frequency ω) are in thermal equilibrium at temperature T.

(a) (3 pts.) Calculate the system partition function.

(b) (4 pts.) Find the system average energy.

(c) (3 pts.) Find $\langle(\Delta E)^2\rangle$, the fluctuations in the system energy.

Problem 8 (10 points)

One mole of ideal monatomic gas undergoes a cycle as shown in the figure below.
(a) (3 pts) Calculate the net work for a single cycle.
(b) (3 pts) What is the difference in the internal energy between state C and A.
(c) (4 pts) Calculate the heat generated when the system goes from $A \to B \to C$.
Note: You might find the following integral helpful

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2}(x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a}) + C \tag{3}$$

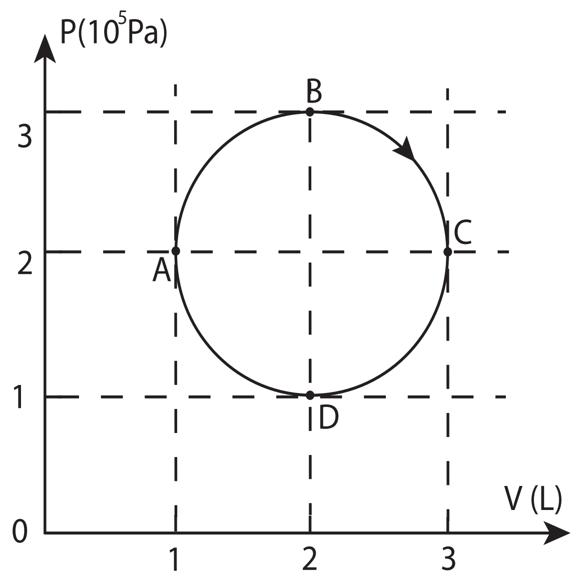

FIG. 1. Thermodynamic cycle.

Problem 9 (10 points)

A K^0 meson has a mass of 498 MeV/c^2 and decays into two charged pions, $K^0 \rightarrow \pi^+ + \pi^-$. The π^+ and π^- mesons have opposite electrical charges but the same mass of 140 MeV/c^2.

If the K^0 meson decays at rest, use energy and momentum conservation to determine the energy, momentum, and speed of either pion.

a) (4 Points) What is the energy (in MeV) of either pion?

b) (3 Points) What is the momentum (in MeV/c) of either pion?

c) (3 Points) What is the speed (in m/s) of either pion?

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Problem 10 (10 points)

Consider a particle of mass m in the potential $U(x) = -\alpha\delta(x)$.

(a) (3 points) Find the general solution of the Schrodinger equation for energies corresponding to the bound state(s).

(b) (3 points) Determine the energy of the ground state.

(c) (4 points) If the parameter α suddenly increases 2 times, what is the probability for the particle to become free?

Problem 11 (10 points)

Consider an electron in the spin state

$$\chi = A \begin{pmatrix} 4 \\ 3i \end{pmatrix}. \tag{4}$$

(a) (2 points) Determine the normalization constant

(b) (3 points) Find the expectation values for all three components of the spin

(c) (3 points) Find the standard deviations for all three spin components

(d) (2 points) Check the uncertainty relation $\Delta s_x \, \Delta s_y$

Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{5}$$

Problem 12 (10 points)

Consider particles of mass m and energy E incident from the left onto potential

$$U(x) = \begin{cases} 0, \ x < 0 \\ V_0, \ x > 0 \end{cases}, \tag{6}$$

where $V_0 > 0$. For all three questions below discuss both cases $E < V_0$ and $E > V_0$.

(a) (4 points) Find the general solution of the Schrodinger equation for $x < 0$ and $x > 0$ regions. Apply boundary/matching conditions.

(b) (3 points) Calculate the probability current density for the incident and reflected waves, as well as in the region $x > 0$.

(c) (3 points) Find the reflection and transmission coefficients as a function of E