PH. D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

EXAMINATION I
CLASSICAL MECHANICS AND THERMAL PHYSICS

MONDAY, AUGUST 25, 2003
9:00 A.M. TO 1:00 P.M.

ROOM 245, PHYSICS RESEACH BUILDING

This examination consists of **four** problems. Do all the problems using a separate booklet for each problem. On the front cover of each booklet, you **must write the following information clearly**:

i) Your special ID number that you obtained from Delores Cowan.
ii) The title of the examination (Classical Mechanics and Thermal Physics).
iii) The problem number that is worked in the booklet (*e.g.* #2).
iv) Please press hard and make your answer legible to read.

You must **NOT** write your name anywhere on the booklet.

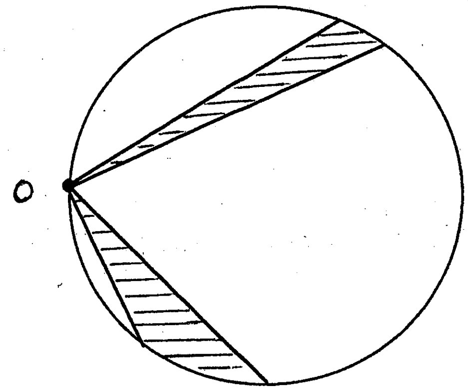
EXAM #1, PROBLEM #1

It is well known that the inverse-square law and the central nature of the gravitational force were based on Kepler's Laws of planetary motion. Now assume that we are observing a particle of mass m moving in a closed orbit under the influence of a different force. Let us study the nature of this force and its potential by making a few observations.

(a) (2 points) First, it is observed that the rate at which of area is swept out by the position vector of the particle is constant in time when the origin is located at point O on the circular orbit. What property of the force can you infer from this observation?

(b) (3 points) Next, the orbit is observed to be a circle of radius R. The period is measured to be T. Based on the known parameters, calculate the maximum and minimum speed of the particle during its periodic motion. At what locations do they happen?

(c) (5 points) Derive the force acting on the particle, $\mathbf{F}(\mathbf{r})$, and its potential function $V(\mathbf{r})$.

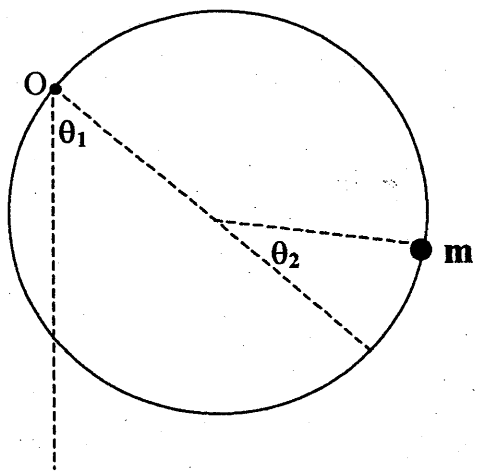


Exam #1, PROBLEM #2

A circular loop of wire of uniform mass density has a radius R and is hung vertically in the x-y plane with one point of the loop as the pivot point. The loop has a total mass M. The gravitational force is in the -y direction. A point-size bead of mass m is restricted to slide without friction on the loop, as shown in the figure below. We consider small-angle rotational motions of the loop and the bead system in the x-y plane.

(a) (4 points) Construct the Lagrangian for the system of the loop and the bead. Obtain the equations of motion.

(b) (6 points) Solve the equations to obtain the normal-mode frequencies of small oscillations. Describe the nature of the normal modes.



EXAM #1, PROBLEM #3

Consider a gas system. Its pressure P, the temperature T, and the specific volume (the volume of the gas divided by the number of the gas particles) v, obey the equation of state:

$$P = \frac{RT}{v-b} - \frac{a}{v^2}$$

Here a and b are positive constants, and R is the usual gas constant.

(a) (2 points) Calculate the isothermal compressibility K_T and the thermal expansion coefficient β.
(b) (6 points) Show that the specific heat C_v (the heat capacity per particle) is a function of temperature alone, *i.e.* it is independent of the specific volume v.
(c) (2 points) Construct the entropy function $S(T,v)$ from the equation of state and C_v.

EXAM #1, PROBLEM #4

Consider a simple model of a long polymer in which a polymer is made of a string of N connected monomers, each of which can be in a compact state or an extended state. A monomer has energy ε_e and length L_e when it is in the extended state, and energy ε_c ($> \varepsilon_e$) and length L_c ($< L_e$) when it is in the compact state.

(a) (2 points) In thermal equilibrium, what is the longest (thermal-equilibrium) length? What is the shortest (thermal-equilibrium) length? Give a brief explanation for each situation.
(b) (3 points) Calculate the entropy as a function of the energy of the polymer.
(c) (5 points) Calculate the length of the polymer as a function of temperature T. Does your calculated result agree with that of (a)?

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EXAMINATION II
ELECTROMAGNETISM and OPTICS

WEDNESDAY, AUGUST 27, 2003
9:00 A.M. TO 1:00 P.M.

ROOM 245, PHYSICS RESEACH BUILDING

This examination consists of **five** problems. Do all the problems using a separate booklet for each problem. On the title page of each booklet, you **must write the following information clearly**:

i) Your special ID number that you obtained from Delores Cowan.
ii) The title of the examination (Electromagnetism and Optics)
iii) The problem number of the problem that is worked in the booklet (*e.g.* #2).
iv) Please press hard to make your answers legible to read.

You must **NOT** write your name anywhere on the booklet.

EXAM #2, PROBLEM #1

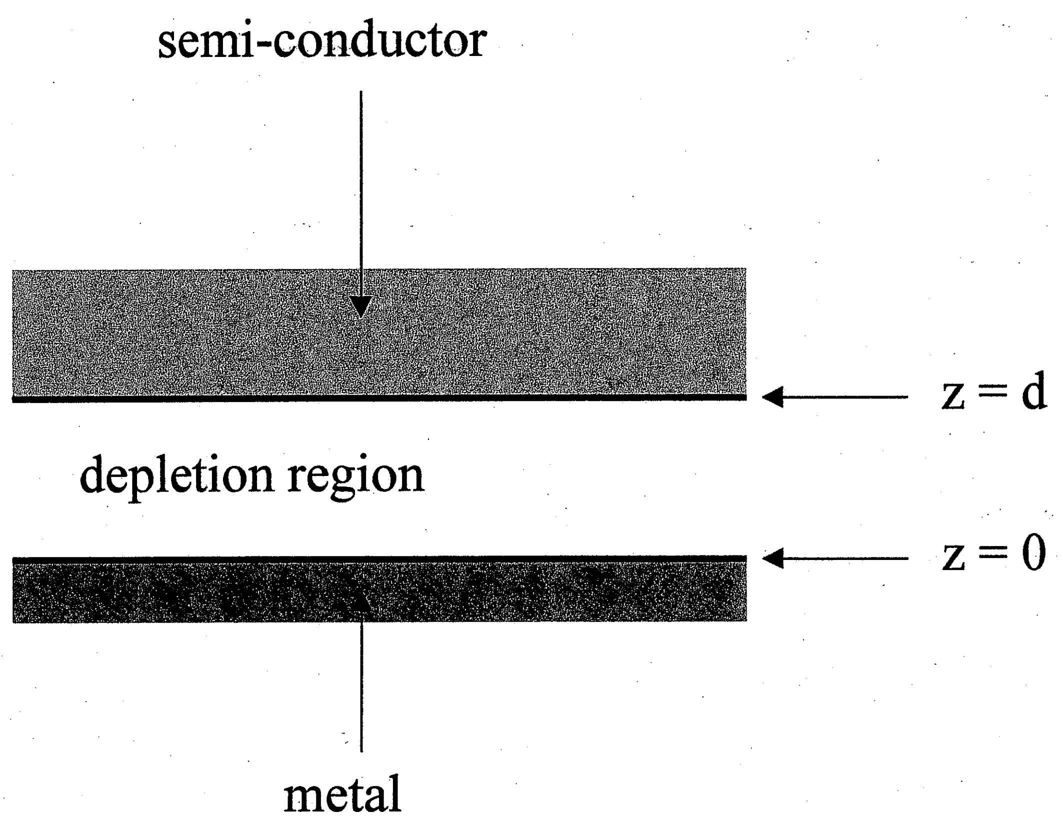
From the point of view of electrostatics, a reverse-biased metal-semiconductor junction can be simply modeled as two conducting regions separated by a layer of immovable background charges. One of the conducting regions is the metallic film, held at some positive potential V. The opposite conducting region is the bulk semiconductor at zero potential. The region in between is the depletion layer in the semi-conductor, where the mobile charge carriers have been pushed away by the bias voltage.

(a) (3 points) Calculate the electric field in the depletion region, assuming that it contains a constant charge density ρ. Choose $z = 0$ at its interface with the metal and assume that $E(d) = 0$, *i.e.* the field reaches zero at the other boundary of the depletion layer.

(b) (3 points) Calculate the potential Φ in the depletion layer. Assume $\Phi(0) = V$ and $\Phi(d) = 0$. Find the relation between V, d, and ρ.

(c) (2 points) Assume that the metal film has area A. What is the dynamic capacitance C (= dQ/dV) of the junction?

(d)(2 points) Metal-semiconductor junctions can be used as variable capacitors. From your calculations can you explain why?



EXAM #2, PROBLEM #2

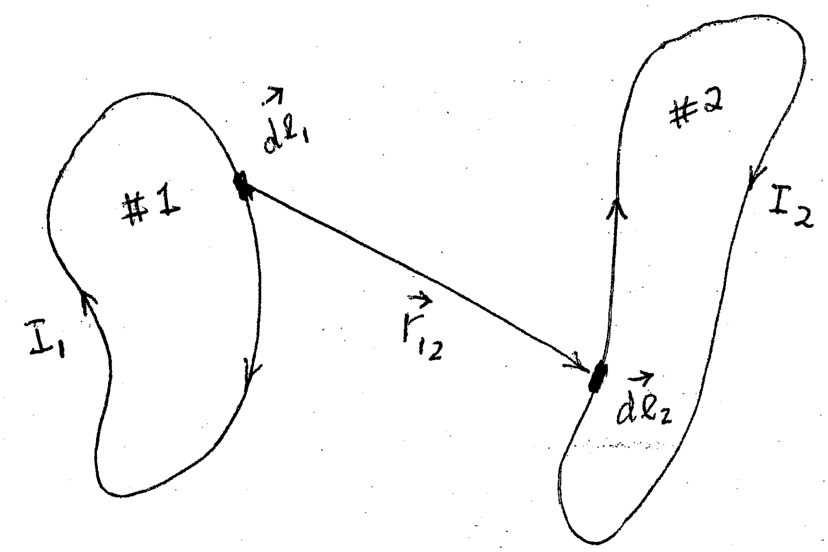
Consider two current loops #1 and #2 carrying current I_1 and I_2, respectively, as shown in the figure below. Let $d\vec{\ell}_1$ and $d\vec{\ell}_2$ be current elements in #1 and #2 and $\vec{r}_{12}$ be the vector leading from $d\vec{\ell}_1$ to $d\vec{\ell}_2$.

(a) (2 points) Use the Biot-Savart law to express the differential magnetic field $d\vec{B}$ at the position of $d\vec{\ell}_1$ due to $d\vec{\ell}_2$. Then express the differential force $d\vec{F}$ experienced by $d\vec{\ell}_1$ due to $d\vec{\ell}_2$.

(b) (2 points) Show by counter example that the differential form of force does not obey Newton's third law of action and reaction.

(c) (3 points) Show that the force of one current loop on another current loop, *i.e.*, the above force integrated over both loops, obeys Newton's third law explicitly.

(d) (3 points) Show that the torque on current loop #1 due to the magnetic force of loop #2 has the same magnetude and opposite direction as that on current loop #2 due to the magnetic force of loop #1.



EXAM #2, PROBLEM #3

(a) (4 points) Two infinite equal and opposite line charges are situated at $x = \pm a$, $y = 0$. The charge per unit length for the two lines are $\pm\lambda$. Calculate the potential $\phi(x, y, z)$ at an arbitrary point in space and show that the equi-potential surfaces are circular cylinders.

(b) (6 points) A twin-wire transmission line can be thought of as two parallel circular wires of radius r separated by a distance d. Use the facts learned from part (a) to calculate the capacitance per unit length of the twin-wire transmission line.

EXAM #2, PROBLEM #4

Consider a dielectric medium in which the electric permittivity varies in space in such a way that the macroscopic Maxwell equations still hold.

(a) (3 points) Eliminate the ***B***-fields from Maxwell equations and derive the differential equation satisfied by the vector ***E***-field.

(b) (7 points) Specialize to the situation in which the electric permittivity is a function of just one coordinate, say z.

$$\varepsilon = \varepsilon(z)$$

Derive the differential equation satisfied by E_x, E_y, *and* E_z and show that at least one of them is decoupled from the other components. Comment on the significance of this decoupling.

EXAM #2, PROBLEM #5

An observer sees an electromagnetic plane wave of frequency $\omega = ck$ (with wave vector $\boldsymbol{k}$ in the x-y plane at an angle θ with respect to the positive x direction) impinging on a moving plane mirror. The mirror is parallel to the y-z plane, and is moving in the x direction with speed v.

(a) (3 points) What is the Lorentz transformation between the observer's rest frame and the mirror's rest frame?

(b) (7 points) Calculate the wavevector and the frequency of the observed reflected wave.

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EXAMINATION III
QUANTUM MECHANICS

FRIDAY, AUGUST 29, 2003
9:00 A.M. TO 1:00 P.M.

ROOM 245, PHYSICS RESEACH BUILDING

This examination consists of **four** problems. Do all the problems using a separate booklet for each problem. On the title page of each booklet, you **must write the following information clearly**:

i) Your special ID number that you obtained from Delores Cowan.
ii) The title of the examination (Quantum Mechanics).
iii) The problem number of the problem that is worked in the booklet (*e.g.* #2).
iv) <u>Please press hard to make your answers legible to read.</u>

You must **NOT** write your name antwhere on the booklet.

EXAM #3, PROBLEM #1

Consider first a particle of mass *m* in one-dimensional space. The potential field is

$$\begin{aligned} V(x) &= 0 && \text{for } x < -a \\ &= -V_0 && \text{for } -a < x < a \\ &= 0 && \text{for } x > a. \end{aligned}$$

Here V_0 and *a* are positive constants.

(a) (2 points) Show that all eigenwavefunctions of the Hamiltonian can have definitive parity. Give a brief argument to show that if the ground state is bound, its parity is even.

(b) (3 points) Find the equation that the eigenenergy of a even-parity bound state has to satisfy. Using this condition, show that there always exists at least one bound state. (You do not have to calculate the ground state energy.) Find the relation between the parameters V_0 and *a* which allows only one even-parity bound state.

Now we consider a particle of mass *m* in three dimensions. The potential is central and has the form:

$$\begin{aligned} V(r) &= -V_0 && \text{for } r < a \\ &= 0 && \text{otherwise} \end{aligned}$$

(c) (5 points) Show that for a fixed value of *a*, there is no bound state unless V_0 is large enough. Find the condition that allows at least one bound state. (Hint: the ground state is an S state.)

EXAM #3, PROBLEM #2

Consider a particle of mass m in a simple harmonic potential

$$V(x) = \frac{1}{2} k x^2$$

Here k is a positive constant.

(a) (1 point) What are the eigenenergies of the particle?

Now, a perturbative potential

$$V'(x) = \alpha x^4$$

is applied. Here α is a positive constant.

(b) (3 points) Calculate the change in the ground state eigenenergy using first-order perturbation theory.

(c) (6 points) Calculate the change in the ground state eigenenergy using second-order perturbation theory.

Formulae for the one-dimensional simple harmonic oscillator that you may find useful:

$$<n'|x|n> = \sqrt{\hbar/(2m\omega)} \left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\right)$$

and

$$<n'|p|n> = -i\sqrt{\hbar m\omega/2} \left(\sqrt{n}\,\delta_{n',n-1} - \sqrt{n+1}\,\delta_{n',n+1}\right)$$

EXAM #3, PROBLEM #3

(a) (3 points) Consider the scattering of non-relativistic particles by a step-down potential

$$V(x) = 0 \qquad \textit{for } x < 0 \textit{ and}$$
$$= -V_0 \qquad \textit{for } x > 0$$

in one dimension, where V_0 is a positive constant.
Assuming that the incident particles are coming from $x = -\infty$, calculate the transmission coefficient as a function of energy, and show that it is less than one for any (positive) finite incident energy.

(b) (7 points) Now consider the scattering when an additional step in the potential function is present at $x = a > 0$, so that the potential is changed to

$$V(x) = 0 \qquad \textit{for } x < 0$$
$$= -V_0 \qquad \textit{for } 0 < x < a$$
$$= 0 \qquad \textit{for } a < x$$

Find the condition that the energy of the incident particle has to satisfy in order for the reflection coefficient to vanish.

Exam #3, Problem #4

Consider a stationary electron in a uniform magnetic field. Electrons are spin $\frac{1}{2}$ particles and hence have intrinsic magnetic moment

$$\boldsymbol{\mu} = -\mu_0 \frac{2}{\hbar} \boldsymbol{S}$$

Here μ_0 is a positive constant and $\boldsymbol{S}$ is the spin of the electron. The Hamiltonian has the form:

$$H = -\boldsymbol{\mu} \bullet \boldsymbol{B}$$

Use the states α and β, which represent respectively the spin up state and spin down state in the z direction, as the basis states to answer the following.

(a) (2 points) A constant magnetic field

$$\boldsymbol{B} = B_0\, \hat{z}$$

is applied. What are the eigenenergies and the corresponding eigenfunctions?

(b) (3 points) Now consider the case when an additional time-dependent field is added. This additional field has constant strength B_1 and rotates in the x-y plane with a constant angular frequency ω, so that

$$\boldsymbol{B} = B_0\, \hat{z} + B_1 [\cos(\omega t)\, \hat{x} + \sin(\omega t)\, \hat{y}]$$

Calculate the matrix elements of the Hamiltonian using α and β as the basis states.

(c) (5 points) The electron is known to be in the spin-down state at time $t = 0$. Calculate the probability of finding it in the spin-up state as a function of time for $t > 0$.