Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

EXAMINATION I
MECHANICS, THERMODYNAMICS AND STATISTICAL PHYSICS

MONDAY, AUGUST 29, 2005
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains four problems. You are to solve all four using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. problem #1, exam #1)
3) Please press hard and make your answers legible.

Please do NOT write your name on the cover or anywhere else in the booklet!

EXAM #1, PROBLEM 1

N weakly-coupled particles obeying Maxwell-Boltzmann statistics may each exist in one of the 3 non-degenerate energy levels of energies $-E$, 0, $+E$. The system is in contact with a thermal reservoir at temperature T.

a. What is the entropy of the system at T=0 K? (2 pts)
b. What is the maximum possible energy of the system? (2 pts)
c. What is the minimum possible energy of the system? (2 pts)
d. What is the partition function of the system? (2 pts)
e. What is the energy of the system as T tends to infinity? (2 pts)

EXAM #1, PROBLEM 2

A massless spring of rest length l_0 (with no tension) has a point mass m connected to one end and the other end is fixed to the ceiling. When fixed to the ceiling, the spring is then subject to gravity and is free to swing like a pendulum. Assume that the spring is stiff and can only stretch along its length, and that the motion of the system is only in one vertical plane.

a. Write down the Lagrangian. (3 pts)

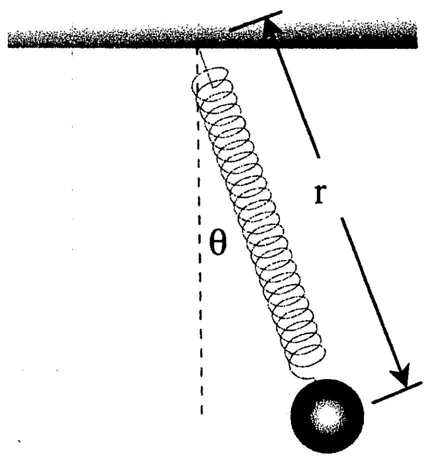
b. Find the Lagrange equations using θ and $\lambda = (r - r_0)/r_0$, where r_0 is the rest length (hanging with mass m). Use $\omega_s^2 = k/m$ and $\omega_p^2 = g/r_0$. (3 pts)

c. Discuss the lowest order approximation to the motion when λ and θ are small with the initial conditions

$$\theta = 0, \quad \dot{\lambda} = 0, \quad \lambda = A, \quad \dot{\theta} = \omega_p B \quad \text{at} \quad t = 0,$$

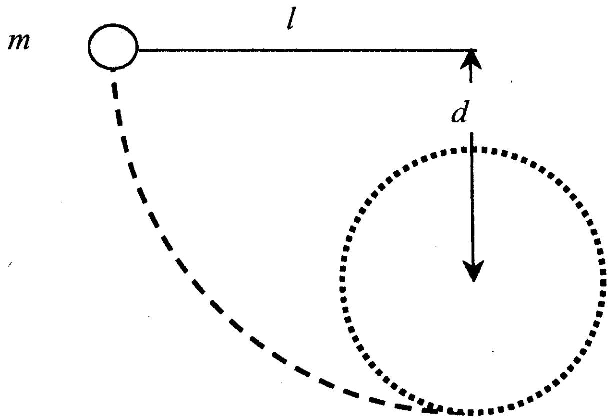
for A and B constants. (2 pts)

d. Discuss the next order approximation to the motion. Under what conditions will the λ motion resonate? Can this be realized physically? (2 pts)



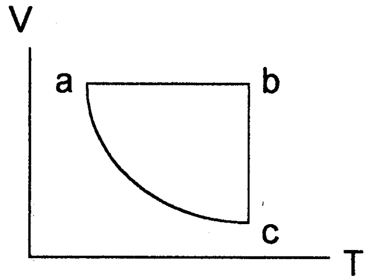
EXAM #1, PROBLEM 3

A pendulum of mass m and length l is released in a horizontal position. A nail a distance d below the pivot causes the mass to move along the path indicated by the dotted line. Find the minimum distance d in terms of l such that the mass will swing completely around in a circle.



EXAM #1, PROBLEM 4

Compressing the system represented in the figure below (note: it is not a *P*-*V* diagram) along the adiabatic path *a*-*c* requires 1000 J. Compressing the system along *b*-*c* requires 1500 J, but 600 J of heat flow out of the system. Calculate the work done, the heat absorbed, and the internal energy change of the system in each process (*a*-*b*, *b*-*c*, *c*-*a*) and in the total cycle *a*-*b*-*c*-*a*.



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EXAMINATION II
ELECTRICITY AND MAGNETISM, OPTICS AND SPECIAL RELATIVITY

WEDNESDAY, AUGUST 31, 2005
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains four problems. You are to solve all four using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

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EXAM#2, PROBLEM 1

1. An insulating ring of radius R has total fixed charge +Q distributed uniformly on its surface.

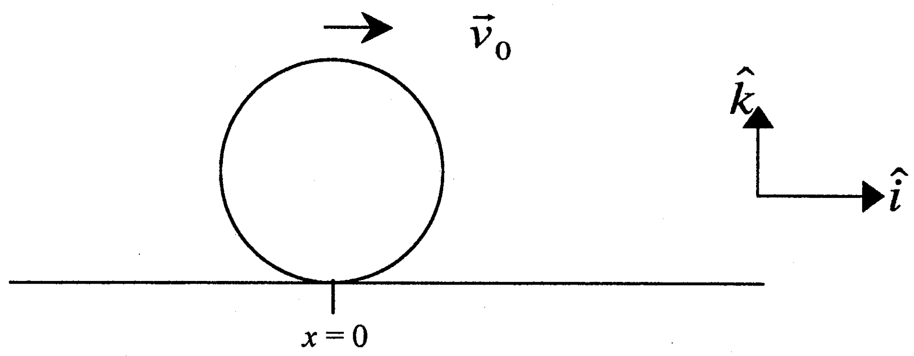
a. What are the electric field and potential at the center of the ring? (3 pts)

b. Find the electric field and potential along the axis of the ring. (4 pts)

c. A point charge –Q of mass M is constrained to slide along the axis of the ring. Show that this charge will exhibit simple harmonic motion for small displacements along the ring axis and find its frequency of oscillations. (4 pts)

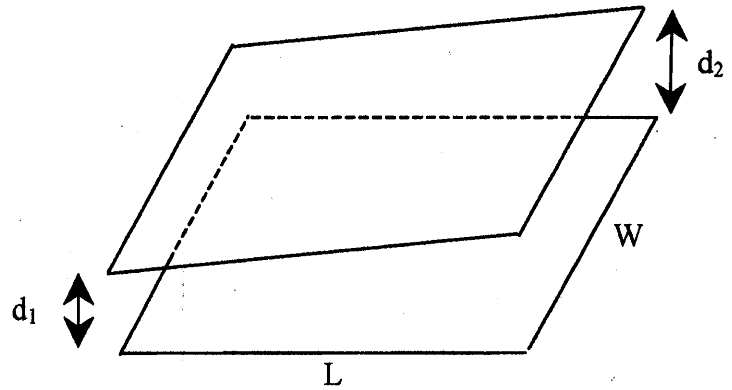
EXAM#2, PROBLEM 2

A conducting ring of mass M, radius a, and resistance R rolls in the x-z plane without slipping over a flat horizontal surface. The region is filled with a magnetic field $\vec{B} = (bx)\hat{j}$, where b is a constant and x is the position. Assume that the ring rolls in the positive x direction with an initial velocity $v_0\hat{i}$, starting with its center at $x = 0$. How far does the ring roll before it stops?



EXAM#2, PROBLEM 3

A capacitor is constructed using two identical rectangular conducting plates of width W and length L. The plates are not parallel. One pair of edges of width W is a distance d_1 apart, while the other is a distance d_2 apart, with $d_1 < d_2$.



a. Neglecting edge effects, when a voltage difference V is placed across the two conductors, find the potential everywhere between the plates. (6 pts)

b. Determine the capacitance. (4 pts)

EXAM#2, PROBLEM 4

A plane electromagnetic wave of intensity I is incident on a semi-infinite glass slab of index of refraction n. The wave vector is at right angles to the surface (normal incidence).

a. (6 pts) Show that the coefficient of reflection (of the intensity) for a single interface at normal incidence is

$$R = \left(\frac{n-1}{n+1}\right)^2$$

b. (4 pts) Calculate the radiation pressure acting on the plate in terms of I and n.

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EXAMINATION III
QUANTUM PHYSICS

FRIDAY, SEPTEMBER 2, 2005
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

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EXAM #3, PROBLEM 1

Quantum phenomena are often negligible in the "macroscopic" world. Show that this is true by providing numerical estimates of the following quantities:

a. The amplitude of the zero-point oscillation for a pendulum of length $l = 1$ m and mass $m = 1$ kg. (4 pts)

b. The tunneling probability for a ball of mass $m = 1$ kg moving at a speed of 1 m/s against a rigid obstacle of height $H = 1$ m and width $W = 1$ m. (3 pts)

c. The width of the diffraction peak for a ball of mass m = 1 kg moving with a velocity 1 m/s through an open window of 1 m × 1 m. (3 pts)

EXAM#3, PROBLEM 2

A particle of mass m is constrained to move between two concentric impermeable spheres of radii $r = a$, and $r = b$. There is no other potential.

a. Write down the Schrödinger equation. What boundary conditions must the wave function satisfy? (4 pts)

b. Find the ground state energy and wave function. (4 pts)

c. What is the pressure on the wall at $r = b$ when the particle is in the ground state? (2 pts)

EXAM#3, PROBLEM 3

Suppose an electron is in a state described by the wave function

$$\psi = \frac{1}{\sqrt{4\pi}}\left(e^{i\phi}\sin\theta + \cos\theta\right)g(r)$$

where

$$\int_0^\infty |g(r)|^2 r^2 dr = 1,$$

and ϕ, θ are the azimuthal and polar angles respectively.

a. What are the possible results of a measurement of the z-component of the orbital angular momentum L_z of the electron in this state? (3 pts)

b. What is the probability of obtaining each of the possible results in part a? (4 pts)

c. What is the expectation value of L_z? (3 pts)

EXAM#3, PROBLEM 4

A rod of length L and uniform mass distribution is pivoted at its center and constrained to rotate in the x-y plane. The rod has mass M. Opposite charges $+Q$ and $-Q$ are fixed to the ends of the rod.

a. Obtain the Hamiltonian, eigenvalues and eigenfunctions for this system. (4 pts)

b. What are the eigenvalues and eigenfunctions to first order in E when a constant weak electric field E is applied in the x direction? (3 pts)

c. Find the approximate energy and wave function of the ground state when the applied electric field is very strong, i.e., when $QEL >> \hbar^2 / ML^2$. (3 pts)