# Ph.D. QUALIFYING EXAMINATION
# DEPARTMENT OF PHYSICS AND ASTRONOMY
# WAYNE STATE UNIVERSITY

## PART I

MONDAY, MAY 8, 2006
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

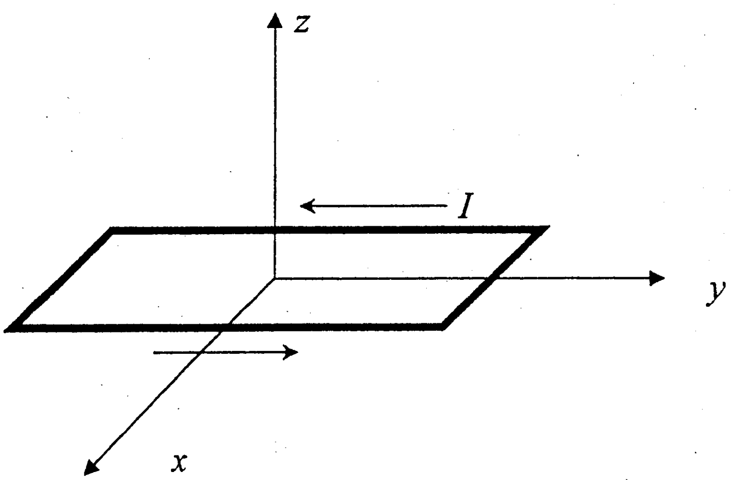
1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. problem #1, part #1)
3) Please press hard and make your answers legible.

**Please do NOT write your name on the cover or anywhere else in the booklet!**

1. A square wire loop of size $2a \times 2a$ lies in the $xy$ plane with its center at the origin and sides parallel to the $x$ and $y$ axes. A counterclockwise current $I$ flows around the loop.

a. (2 pts) Sketch the magnetic field at any point on the $z$ axis by indicating the magnitude and direction of the field at enough points to illustrate the trend of variation of the field with $z$ for both $z > 0$ and $z < 0$.

b. (8 pts) Calculate the magnetic field at any point on the $z$ axis.



2. The lower 15 km of the atmosphere – the troposphere – is often in a steady state so that $PV^{\gamma}$ is independent of the altitude, where $\gamma = C_p / C_V$. In the following, assume that air is an ideal gas consisting of a single species of diatomic molecule.

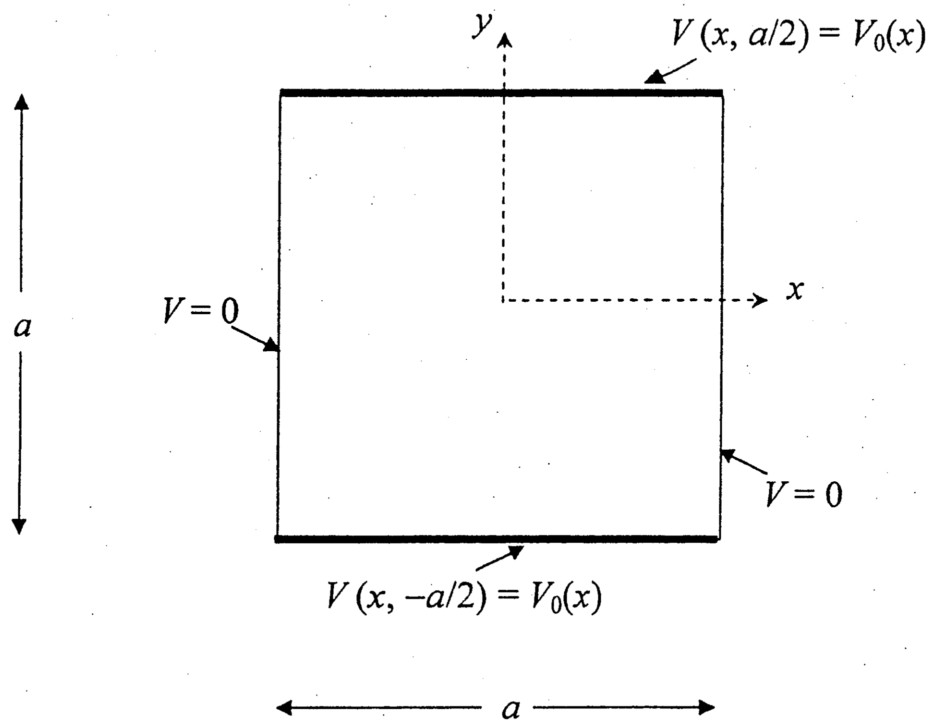
a. (2 pts) Write down the specific heats $C_v$, $C_p$ and obtain $\gamma = C_P / C_V$.

b. (6 pts) Pressure varies with altitude $z$ as $P = P_0 - \rho g z$, where $\rho$ is the mass density and $P_0$ is the pressure at sea level. Find the rate of change of temperature in this model with altitude $dT/dz$.

c. (2 pts) Assume that the average diatomic molecule of air has a molar mass $\mu = 29$ g/mole and that the gas constant is $R = N_{av}k_B = 8.31$ J deg$^{-1}$ mole$^{-1}$. Estimate the temperature at the top of Mt. Everest, which is about 8,850 m above sea level. Assume that the temperature is 25 °C at sea level.

3. A square $a \times a$ pipe of infinite length centered at the origin has two opposite sides grounded while the other sides at $y = \pm a/2$ are maintained at the potential $V_0(x) = C\cos(\pi x / a)$, as shown in the figure.

a. (6 pts) Find the potential everywhere inside the square.
b. (4 pts) Find the surface charge density on the surfaces at $x = \pm a/2$.



4. A particle of mass m is confined to a region $0 < x < a$ in one dimension by infinitely high walls at $x = 0$ and $x = a$. At $t = 0$ the particle is initially in the left half of the well such that:

$$\Psi(x,0) = \begin{cases} \sqrt{2/a}, & 0 \leq x < a/2 \\ 0, & a/2 \leq x \leq a \end{cases}.$$

a. (2 pts) Write down the eigenfunctions and energy eigenvalues describing the bound states of the infinite square well potential for $0 < x < a$.

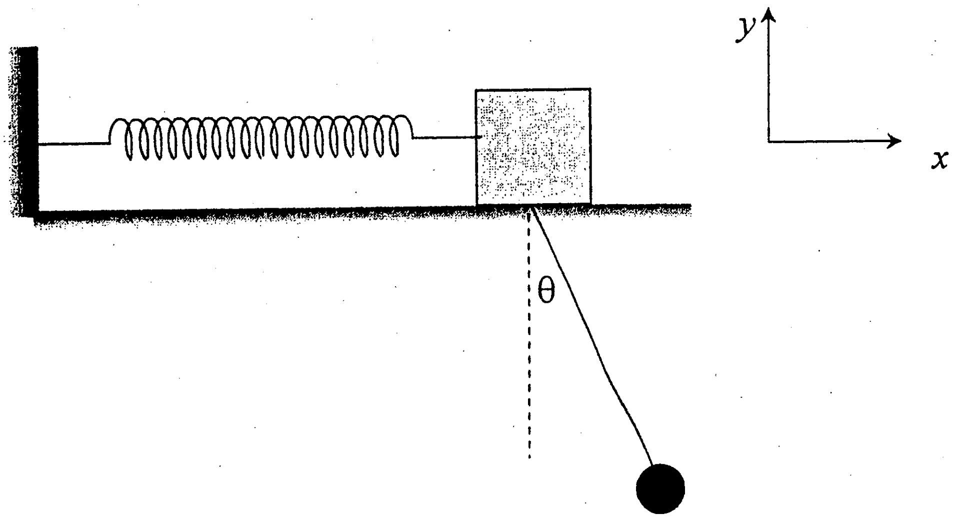
b. (6 pts) Find the time-dependent wave function $\Psi(x,t)$ as an expansion in square well eigenfunctions.

c. (2 pts) What are the probabilities that measurements will find the particle in the square-well eigenstate corresponding to the first and second excited states?

5. A simple pendulum consisting of a mass $m$ and a weightless string of length $L$ is mounted on a support of mass $M$. The support is attached to a horizontal spring with force constant $k$ as shown in the figure.

a. (7 pts) Set up the Lagrange equations for this system.

b. (8 pts) Find the frequencies of small oscillations, assuming that both $\theta$ and the generalized velocities of both masses are small.



6. Consider an ideal gas of $N$ non-relativistic identical bosons of mass $m$, energy $\varepsilon = p^2/2m$, and spin zero in a volume $V$ at a temperature $T$.

a. (2 pts) Write the Bose-Einstein distribution in terms of $\varepsilon$, $T$ and the chemical potential $\mu$.

b. (5 pts) Argue that $\mu \leq 0$ for bosons in general, and that $\mu = 0$ at the critical temperature $T_c$ where Bose-Einstein condensation occurs.

c. (5 pts) Determine the critical volume $V_c$ below which condensation occurs in three dimensions. You may leave the answer in terms of an undetermined numerical factor; we are only interested in how the result depends on $m$, $T$, $N$, $\hbar$, and Boltzmann's constant $k$.

d. (3 pts) What is the critical volume in two dimensions?

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## PART II

WEDNESDAY, MAY 10, 2006
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

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1. Consider the classical motion of a mass m under the influence of the force $\vec{F} = -k\vec{r}$, where $k$ is a positive constant and $\vec{r}$ is the position vector of the particle.

a. (4 pts) Find the position of the particle as a function of time, assuming that at time $t = 0$, $x = a$, $y = 0$, $z = 0$, $v_x = 0$, $v_y = V$, and, $v_z = 0$. *Hint:* you don't need to use the radial orbit equation.

b. (2 pts) Show that the orbit is an ellipse.

c. (2 pts) Find the period.

d. (2 pts) What is the relation between the period of the orbit and the semimajor axis of the ellipse? Does the motion obey Kepler's third law of planetary motion?

**2.** Positronium consists of an electron $e^-$ and a positron $e^+$, each of mass $m$ in a bound state. You may neglect the binding energy in the following. Suppose that the positronium annihilates into two photons.

a. (5 pts) Calculate the energy, momentum, and frequency of the photons in the reference frame where the positronium is at rest.

b. (5 pts) Now suppose that the positronium is moving with velocity $v$ away from an observer when it annihilates, such that one of the photons moves directly toward the observer. Find the frequency of this photon as measured by the observer in terms of the frequency in the positronium rest frame.

3. An atom moves above an impenetrable surface. It is attracted to the surface by the potential

$$V(x, y, z) = \begin{cases} kz^2/2; & 0 < z < \infty \\ \infty; & -\infty < z < 0 \end{cases}$$

where $z$ is the distance from the surface.

a. (2 pts) Write a Schrödinger equation for the energy eigenstates and eigenvalues of the atom. Treat the atom as a point particle.

b. (4 pts) Find the $x$ and $y$ dependence of the eigenstates.

c. (4 pts) Find the energy eigenvalues. Sketch the ground state wave function as a function of $z$ in relation to the potential $V$.

**4.** Two spin ½ particles in one dimension interact through a potential that may be approximated by an infinite square well:

$$V(|x_1 - x_2|) = \begin{cases} 0; & |x_1 - x_2| \le a \\ \infty; & |x_1 - x_2| > a \end{cases}$$

a. (4 pts) Write down the Schrödinger equation for the two-particle system in terms of the coordinates $x = x_1 - x_2$ and $X = (x_1 + x_2)/2$.

b. (3 pts) Assume the total momentum of the pair is zero and that both particles are in the same spin state (up, for example). Write the general form of the two-particle wave function in terms of the spatial wave function and the spinors $\chi_{s_i}$, where $s_i$ is the spin state for particle $i = 1, 2$.

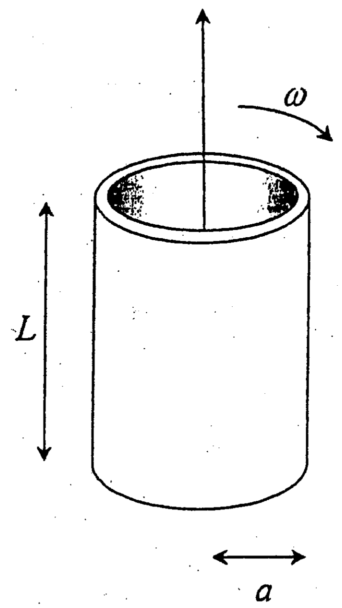
c. (3 pts) Find the lowest energy of the two particles in the eigenstate described in part $b$. How does the spin state of the particles affect the result?

5. A cylindrical thin shell of electric charge has a length $L$ and radius $a$, where $L >> a$. The surface charge density on the shell is $\sigma$. The shell rotates about its axis with an angular velocity $\omega$ that increases with time $t > 0$ as $\omega = kt$, where $k$ is a positive constant. Neglecting fringe and radiation effects, determine the following quantities everywhere inside the cylinder for t > 0:

a. (5 pts) the magnetic field

b. (5 pts) the electric field

c. (5 pts) the total electric and magnetic field energy



6. A free particle of mass $m$ moves in one dimension. It approaches the potential $V(x) = \alpha\delta(x)$ from the left. Suppose that the wave number of the particle is $k$ and that $\alpha > 0$.

a. (3 pts) Write down the general form of the wave function including reflected and transmitted waves.

b. (6 pts) Find the amplitude $t$ of the transmitted wave in terms of $\alpha$, $k$, $m$, and $\hbar$.

c. (6 pts) A neutron interferometer incorporates a beam splitter and mirrors A and B as shown below. The two arms through A and B have equal length. One can vary the relative phase in the two arms by placing a very thin plastic sheet in the beam in one arm of the interferometer. Assume that the effect of the plastic sheet can be described by the delta potential introduced. Find the phase difference of the wave functions between the two arms of the interferometer due to the plastic sheet (ignore the neutron's spin).