Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

TUESDAY, MAY 6, 2008
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

1) Your special ID number that you obtained from Delores Cowen
2) The problem number and the title of the exam (i.e. problem #1, part #1)
3) Please press hard and make your answers legible.

Please do NOT write your name on the cover or anywhere else in the booklet!

1. (10 points) A particle of mass m and electric charge e is initially at rest. It is accelerated for a time t by a uniform electric field E, until it attains a speed comparable with the speed of light c.

(a) (3points) What is the momentum of the particle at the end of the acceleration?

(b) (3points) What is the velocity of the particle at the end of the acceleration?

(c) (4points) After the beam is accelerated to its final momentum, a lifetime measurement is undertaken by watching the surviving particles decay. If the particle lifetime is τ in its rest frame, find the lifetime measured by a stationary observer.

2. (10 points) The initial wavefunction for a one-dimensional simple harmonic oscillator at t=0 is given by

$$\Psi(x,t=0)=\frac{1}{\sqrt{3}}\psi_0(x)+\frac{1}{\sqrt{6}}\psi_1(x)+\frac{1}{\sqrt{2}}\psi_3(x),$$

with ψ_n (n=0, 1, 3) the normalized eigenfunctions of the ground state, the first excited state, and the third excited state. The Hamiltonian can be written as

$$\hat{H}=\frac{\hat{p}^2}{2m}+\frac{1}{2}m\omega^2\hat{x}^2.$$

(a) (2 points) Rewrite the Hamiltonian in terms of ladder operators $\hat{a}_\pm=\left(m\omega\hat{x}\mp i\hat{p}\right)/\sqrt{2\hbar m\omega}$. Derive the energy eigenvalue for any stationary state ψ_n.
(Hint: use $\hat{a}_+\psi_n=\sqrt{n+1}\psi_{n+1}$, $\hat{a}_-\psi_n=\sqrt{n}\,\psi_{n-1}$.)

(b) (1 point) Given the above initial condition, what is the wavefunction $\Psi(x,t)$ at any later time t>0? (Assume that ψ_n is known.)

(c) (7 points) Determine the expectation values (as a function of time t) for the following quantities: the Hamiltonian $\hat{H}$, the kinetic energy, the momentum $\hat{p}$, and the position $\hat{x}$.

3. (10 points) Consider a system of two electrons which interact only through

$$\hat{H} = -\frac{J}{\hbar^2}\left(\hat{S}_x^{(1)}\hat{S}_x^{(2)} + \hat{S}_y^{(1)}\hat{S}_y^{(2)}\right),$$

where J is a constant, and $\hat{\mathbf{S}}^{(i)} = (\hat{S}_x^{(i)}, \hat{S}_y^{(i)}, \hat{S}_z^{(i)})$, with i=1,2, are the spin operators for electrons 1 and 2 respectively.

(a) (6 points) What are the energy eigenvalues of this system? (Hint: first express the Hamiltonian in terms of operators for total spin.)

(b) (4 points) Now apply a magnetic field B along the z direction. Write down the new Hamiltonian. What are the corresponding new energy eigenvalues? (Hint: the gyromagnetic ratio of the electron is $\gamma = -e/m$, with m the mass of electron.)

4. (10 points) A substance obeys the equation of state

$$P = A\frac{T}{V} + \frac{B(T)}{V^2}$$

where A is a constant and $B(T)$ is a function of the temperature T only. The substance is initially at temperature T and volume V_0 and is expanded isothermally and reversibly to volume $V_1 = 2V_0$.

(a) (5 points) Find the work done in the expansion (on the gas).

(b) (5 points) Find the heat absorbed in the expansion.

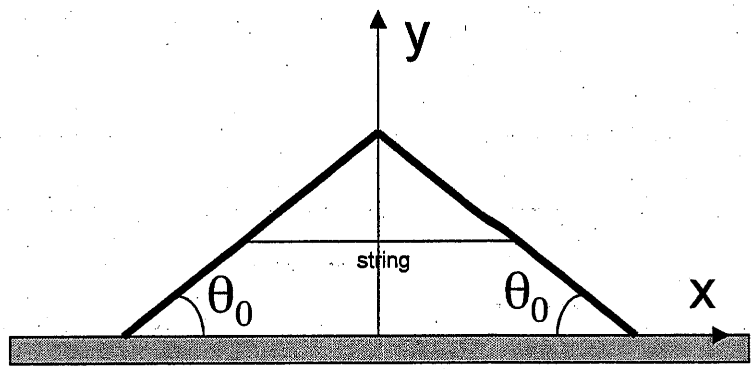
Hint: A Maxwell relation may be useful.

5. (15 points) Two thin beams of mass m and length l are connected by a frictionless hinge and a string. The system rests on a smooth, frictionless surface in the way shown in the Figure. At $t = 0$ the string is cut. In the following you may neglect the string and the mass of the hinge.

(a) (2 point) Given the symmetry of the problem, determine a generalized coordinate to express the state of the system.

(b) (9 points) Determine the speed of the hinge when it hits the floor.

(c) (4 points) Find the time it takes for the hinge to hit the floor, expressing this in terms of a dimensionless integral which you need not evaluate explicitly.



6. (15 points) A linearly polarized EM wave,

$$E_y = Ee^{i(k \cdot z - \omega t)}, \quad E_x = E_z = 0,$$

propagates in a neutral medium with n free electrons per unit volume, all other charges in the problem being fixed and not affecting the wave. The electrons will oscillate as the wave propagates.

(a) (3points) Find an expression for the current density as a function of the wave parameters, assuming small oscillations, no interaction between the electrons, and the electron velocity $v << c$.

(b) (2points) Write Maxwell's equations for the field in the medium, again assuming $v << c$.

(c) (7points) From Maxwell's equations write the differential equations for the spatial dependence of a wave of frequency ω in such a medium.

(d) (3points) Show that such equation can be satisfied by the wave provided that $\omega^2 > (ne^2/m\epsilon_0)$, where m is the electron mass.

Ph.D. QUALIFYING EXAMINATION
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PART II

THURSDAY, MAY 8, 2008
9:00 A.M. — 1:00 P.M.

ROOM 245 PHYSICS BUILDING

INSTRUCTIONS: This paper contains six problems. The first four problems are worth 10 points per problem and the last two are worth 15 points. You are to solve all six using a separate booklet for each problem. On the front cover of each booklet, you must write the following information.

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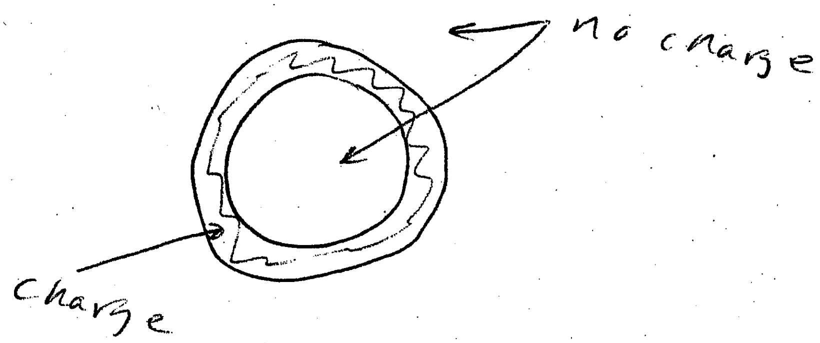
1.(10 points) A static electric charge is distributed in a spherical shell of inner radius R_1 and outer radius R_2. The electric charge density is given by $\rho(r) = a + br$, where r is the distance from center. The electric charge density is zero everywhere outside the spherical shell. The potential $\Phi(r)$ is assumed to be zero at infinite distance. Find

(a) (4points) The electric field everywhere as a function of r.

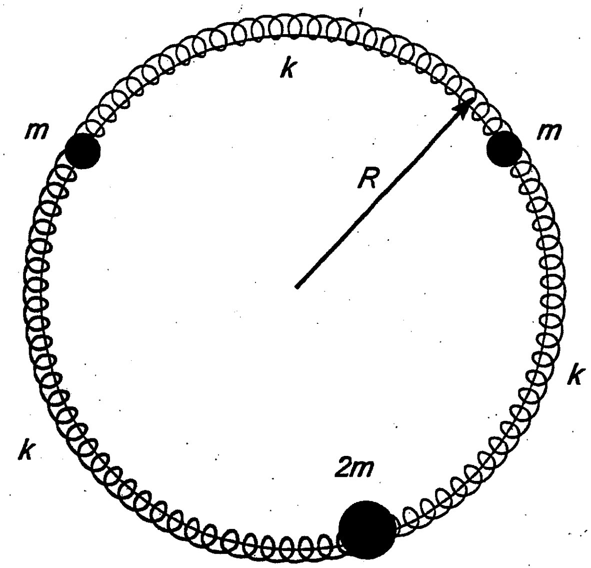
(b) (4points) The electric potential at the origin ($\Phi(0)$).

(c) (2points) The field energy density everywhere.

Provided during the exam:



2. (10 points) Two masses, each of mass m, and a third mass of mass $m_3 = 2m$, are interconnected by identical massless springs of spring constant k and all are placed on a smooth circular hoop as shown in the figure. The hoop is fixed in space, and the energy in the springs is dependent only on their arc lengths. Neglect gravity and friction, treat the masses as point-like, and consider only oscillations involving small changes to the lengths of the springs. Determine the natural frequencies of the system, and the shape of the associated modes of vibration.

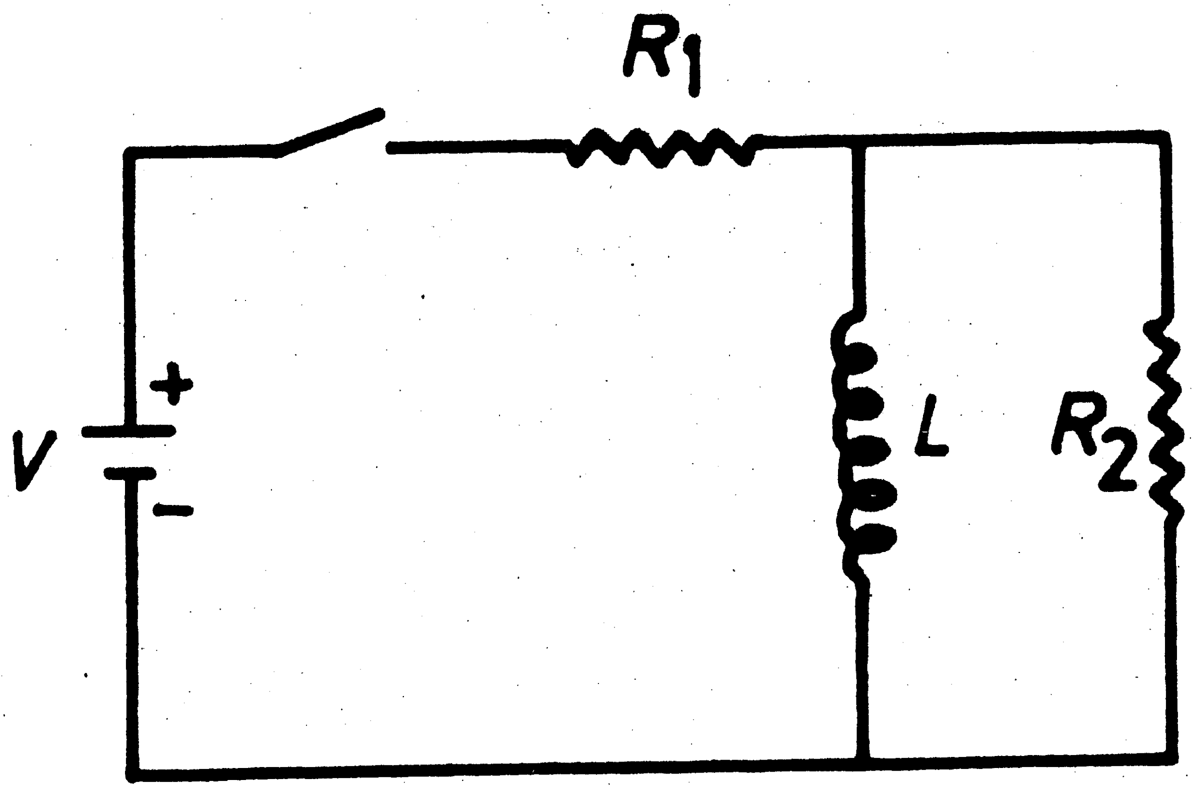


3. (10 points) In the circuit shown below, the resistance of L is negligible and initially the switch is open and the current is zero.

(a) (2points) Write any three independent Kirchoff equations using any node or loop in the figure, when the switch is closed.

(b) (4points) Find the the total quantity of heat dissipated in the resistance R_2 when the switch is closed and remains closed for a long time.

(c) (4points) Find the the total quantity of heat dissipated in the resistance R_2 when the switch, after being closed for a long time, is opened and remains open for a long time.



4. (10 points) For a hydrogenic atom consisting of a single electron with mass m orbiting a nucleus of Z protons, the ground state wavefunction of the electron is

$$\psi(r) = Ae^{-Zr/a},$$

with $A = \sqrt{Z^3/(\pi a^3)}$, $a = 4\pi\varepsilon_0\hbar^2/(me^2)$ the Bohr radius, and r the radial variable in three-dimensional spherical coordinates.

(a) (3 points) Write down the Hamiltonian, and find the ground state energy of the hydrogenic atom (given that the ground state energy for the hydrogen atom (with Z=1) is $E_1 = -\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2$).

(b) (3 points) Calculate the expectation values of the potential energy and the kinetic energy in the ground state.

(c) (4 points) What is the probability $P(r)dr$ of finding the electron between distance r and $r+dr$ from the origin? At what distance r is the probability density $P(r)$ a maximum?

5. (15 points) Imagine the universe to be a cube with sides of length L and impenetrable walls.

   (a) (7 points) Derive the density of states $g(\nu)d\nu$ for photons confined to a cubical cavity, where ν is the frequency.

   (b) (2 points) What is the probability to find a photon with frequency ν in the cavity at temperature T?

   (c) (6 points) If the temperature of the universe is 2.7 K and $L = 10^{28}$ cm, estimate the total number of photons in the universe, and the energy content in these photons.

Some constants:

$$k = 1.381 \times 10^{-23}\ \mathrm{J/K} \qquad h = 6.626 \times 10^{-34}\ \mathrm{J\,s} \qquad c = 2.998 \times 10^{8}\ \mathrm{m/s}$$

and integrals:

$$\begin{aligned}
\int_0^\infty \frac{x\,dx}{e^x - 1} &= \Gamma(2)\zeta(2) = \frac{\pi^2}{6} & \int_0^\infty \frac{x\,dx}{e^x + 1} &= \frac{1}{2}\Gamma(2)\zeta(2) = \frac{\pi^2}{12} \\
\int_0^\infty \frac{x^2\,dx}{e^x - 1} &= \Gamma(3)\zeta(3) \approx 2.404 & \int_0^\infty \frac{x^2\,dx}{e^x + 1} &= \frac{3}{4}\Gamma(3)\zeta(3) \approx 1.803 \\
\int_0^\infty \frac{x^3\,dx}{e^x - 1} &= \Gamma(4)\zeta(4) = \frac{\pi^4}{15} & \int_0^\infty \frac{x^3\,dx}{e^x + 1} &= \frac{7}{8}\Gamma(4)\zeta(4) = \frac{7\pi^4}{120}
\end{aligned}$$

6. (15 points) Consider a particle with mass m and energy $E > V_0 > 0$ coming from $x = -\infty$ towards a potential barrier

$$V(x) = \begin{cases} 0, & \text{for } x < 0 \\ V_0, & \text{for } 0 < x < a \\ 0, & \text{for } x > a \end{cases}$$

(a) (8 points) Calculate the transmission coefficient and also the reflection coefficient.

(b) (3 points) Show the condition of transmission resonance (the Ramsauer-Townsend effect) for which no reflection occurs, and then express this condition in terms of particle wavelength λ inside the barrier (i.e., $0 < x < a$).

(c) (4 points) What are the corresponding results of (a) and (b) if we have $V_0 = -V_0' < 0$ and energy $E > 0$?