Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, MAY 8, 2009
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems. The first four problems are worth 10 points each and the last two are worth 15 points each. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

1) your special ID number that you obtained from Delores Cowen,
2) the problem number and the title of the exam (i.e., Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

Problem 1. (10 points)

If a tunnel could be dug through the center of the Earth along an Earth diameter, one could start digging in Chile and end up in China. Assuming the tunnel is evacuated, a stone dropped from rest in Chile will eventually appear in China. The Earth's radius is R, its mass is M, and the time it takes for the stone to go from Chile to China is T. Assume that the Earth is a perfect sphere, that it has uniform density, and that the rotational motion of the Earth can be neglected.

a) (3 pts.) Write the differential equation for the motion of the stone.

b) (3 pts.) Solve for T as a function of the other given variables.

c) (4 pts.) Assume that the stone is thrown down with initial speed v. Calculate the value of v for which the transit time T is half of its value when $v = 0$, calculated in part b).

Problem 2. (10 points)

An electrical current I flows in a thin, straight conductor extending along the x-axis from $x = 0$ to $x = a$. The goal of this problem is to find the resulting magnetic field $\vec{B}$ at a point located at $(x, y, z) = (0, b, 0)$.

a) (2 pts.) State the Biot-Savart law and use it to determine the direction of $\vec{B}$.

b) (2 pts.) Using the Biot-Savart law, determine the contribution to $\vec{B}$ from a differential length of the wire located at position x.

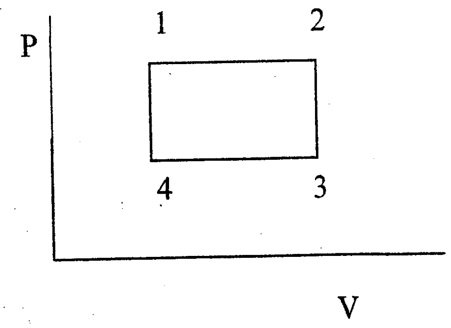
c) (3 pts.) Integrate the expression in b) to obtain $\vec{B}$. Hint: for the integration variable, use the angle between the x-axis and a line extending from $(x, 0, 0)$ to $(0, b, 0)$.

d) (1 pt.) Find $\vec{B}$ in the limit $a \to \infty$.

e) (2 pts.) Use Ampere's law to find the magnetic field at a distance b from a thin, straight conductor of infinite length with current I. Based on superposition and symmetry, use this result to find the answer to d).

Problem 3. (10 points)

Consider a reversible cycle for an arbitrary substance (not an ideal gas) consisting of two isochors and two isobars (see Figure) with constant heat capacities C_V and C_P at constant volume and pressure, and temperatures T_1, T_2, T_3, T_4 at points 1, 2, 3, and 4 respectively.



a) (5 pts.) Calculate the entropy changes along each leg of the process. What is the total change in entropy?

b) (5 pts.) What is the relationship between T_1, T_2, T_3, T_4, for this cycle?

Problem 4. (10 points)

A quantum-mechanical non-relativistic particle of mass m and energy $E > 0$ moves in one dimension. The particle approaches a delta-function potential $V(x) = K\delta(x)$ where K is a non-zero, real constant, $K < 0$ or $K > 0$.

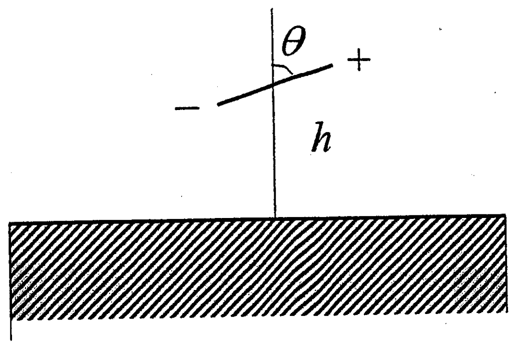
a) (2 pts.) Determine the dimensions (units) of $\delta(x)$ and K.

b) (6 pts.) Find an expression for the transmission probability P as a function of E.

c) (2 pts.) Sketch a graph of P versus E for the two cases $K < 0$ and $K > 0$. For each graph, indicate if there is any value of E for which P takes on the classical value.

Problem 5. (15 points)

An electric dipole p is placed a distance h above a perfectly conducting plane and is kept at an angle θ with respect to the normal to this plane (See Figure).



a) (2 pts.) Indicate the position and the orientation of the image dipole. What is the direction of the force on the dipole?

b) (4 pts.) Derive the potential of an electric dipole at a distance r much larger than the distance between the two charges.

c) (4 pts.) Derive the electric field from a dipole and the interaction energy between the two dipoles.

d) (5 pts.) Using the results of c) find the work needed to remove the dipole to infinity.

Problem 6. (15 points)

Consider the spin state of a spin-1/2 particle.

a) (3 pts.) Find the eigenstates of $S_z = (\hbar/2)\sigma_z$, show that the eigenvalues are $\pm\hbar/2$, and find the corresponding eigenstates.

b) (4 pts.) Another possibility is that the spin angular momentum of the particle points in the $+x$-direction. Find this state in terms of the states found in part a).

c) (4 pts.) The particle is placed in a uniform magnetic field $\vec{B}$, so that the Hamiltonian is given by

$$H = -\gamma\vec{\sigma}\cdot\vec{B} \text{ with } \vec{B} = B\hat{z}$$

and γ is a constant. Suppose that at initial time $t = 0$ the spin wavefunction is what you obtained in part b). What is the wavefunction at a later time $t > 0$?

d) (4 pts.) Calculate the expectation value of $S_i = (\hbar/2)\sigma_i$ as a function of time for $i = x$, y, and z.

Useful information:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Ph.D. QUALIFYING EXAMINATION
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PART II

MONDAY, MAY 11, 2009
9:00 A.M. - 1:00 P.M.

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems. The first four problems are worth 10 points each and the last two are worth 15 points each. Solve all six problems using a separate booklet for each problem. On the front cover of each booklet, write the following information:

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Problem 1. (10 points)

Consider a particle moving in one dimension in a "halved" harmonic oscillator with a potential

$$U(x) = \begin{cases} \infty & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2 & x > 0. \end{cases}$$

The ground state wave function of a standard harmonic oscillator is

$$\psi_0 \sim e^{-(m\omega/2\hbar)x^2}.$$

The creation operator is

$$a^+ = \frac{1}{\sqrt{2m\hbar\omega}}(p + im\omega x).$$

a) (1 pt.) What are the eigenstate energies of the standard harmonic oscillator?

b) (3 pts.) Sketch the first three wave functions of the standard harmonic oscillator and indicate the symmetry corresponding to each of them.

c) (3 pts.) What is the ground state energy for the halved harmonic oscillator system?

d) (3 pts.) What is the ground state wave function for the halved harmonic oscillator potential? (There is no need to normalize the wave function.)

Problem 2. (10 points)

Two infinitely long wires are oriented parallel to the x-axis and carry uniform charge densities $+\lambda$ and $-\lambda$. The wire with $+\lambda$ passes through the point $(y, z) = (a, 0)$ and the wire with $-\lambda$ passes through the point $(y, z) = (-a, 0)$.

a) (7 pts.) Find the potential V at any point (x, y, z) using the potential at the origin as the reference point.

b) (3 pts.) Show that the equipotential surfaces are cylinders, and for a given V locate the axis and the radius of the constant potential cylinder.

Problem 3. (10 points)

A particle of mass m moves non-relativistically under the influence of the potential

$$V = -\frac{\hbar^2}{amr}$$

where a is a positive real constant and r is the radial coordinate. The particle is in the ground state $\Psi_g(r, \theta, \phi, t)$ which has a radial wave function of exponential form.

Note that the Laplacian operator can be written as

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - \frac{L^2}{r^2 \hbar^2}$$

where L^2 is the operator for squared angular momentum.

a) (1 pt.) Evaluate $L^2 \Psi_g$.

b) (6 pts.) Find the normalized, time-independent wavefunction ψ_g in terms of the given constants.

c) (2 pts.) Find the energy of the ground state in terms of the given constants and show that your expression has correct dimensions.

d) (1 pt.) Find $\Psi_g(r, \theta, \phi, t)$ in terms of the given constants.

Problem 4. (10 points)

A very long straight wire of length L and radius R carries a current I as a voltage V is applied to the two ends of the wire.

a) (3 pts.) Find the electric and magnetic fields in the wire.

b) (2 pts.) Find the Poynting vector inside the wire.

c) (5 pts.) Find the energy flux and compare it to the Joule heating. Comment on the significance of the result.

Problem 5. (15 points)

Consider a photon gas in a volume V and at a temperature T.

a) (5 pts.) Calculate the density of states of the photon gas. Define any variables you introduce.

b) (5 pts.) Calculate the energy of the photon gas as a function of T and V. Show that it has the form

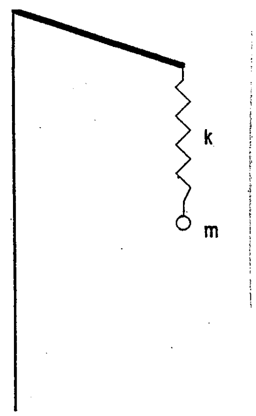
$$U(T, V) = AT^4V$$

where A is a universal constant which you do not need to calculate.

c) (5 pts.) Calculate the pressure exerted by the photon gas on the wall as a function of T and V.

Problem 6. (15 points)

A spring pendulum consists of a mass m attached to a spring of strength k. The mass is free to move in a vertical plane subject to the spring and gravity force. See Figure. When the spring is not attached to any weight, its length is l. The mass can be above or below the pivot point but the spring will not bend.



Determine

a) (2 pts.) the number of degrees of freedom of the system,

b) (5 pts.) the Lagrangian of the system and associated equations,

c) (3 pts.) the equilibrium points and their stability, and

d) (5 pts.) the frequencies of the normal small oscillation modes for the system.