Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

WEDNESDAY, JANUARY 4, 2012
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

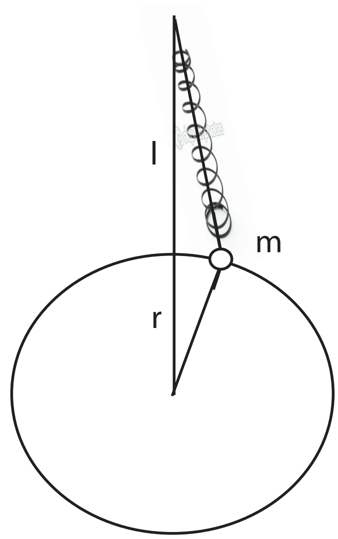
INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** A small bead of mass m can move without friction along a circular wire of radius r. A spring of length ℓ is attached to the bead. The spring exerts a force F on the bead. Assume that the force F is constant. Ignore gravity.



(a) How many degrees of freedom does this system have? (1 pt.)

(b) Write down the Lagrangian for this system. (7 pts.)

(c) Find the frequency of small oscillations, *i.e.* in the limit that the displacement of the bead is much smaller than ℓ, and r. (2 pts.)

2. **(10 points):** Consider a collection of identical noninteracting atoms, each of which has total angular momentum J. The system is in thermal equilibrium at temperature T and is in the presence of an applied magnetic field $\vec{H} = H\hat{z}$. The magnetic dipole moment associated with each atom is given by $\vec{\mu} = -g\mu_B\vec{J}$, where g is the gyromagnetic ration and μ_B is the Bohr magneton.

   (a) For an atom in this system, list the possible values of μ, the magnetic moment along the magnetic field direction and identify the magnetic energy corresponding to each state. (2 pts.)

   (b) Determine the mean value of the magnetic moment μ and the magnetization of the system M for $J = 1$. (6 pts.)

   (c) Find the magnetization of the system in the limits $H \to \infty$ and $H \to 0$ and discuss the physical meaning of the results. (2 pts.)

3. **(10 points):** A particle of mass m is confined in a one-dimensional square-well potential given by

$$V(x) = \begin{cases} -\alpha\delta(x - a/2), & 0 \leq x \leq a \\ \infty, & \text{otherwise} \end{cases}$$

where $\alpha > 0$.

(a) Solve the Schrödinger equation with appropriate boundary conditions and find the transcendental equation determining the allowed energies for $E < 0$. (6 pts)

(b) Sketch the graphical solution to the transcendental equation by plotting both sides of the equation on a figure. (1 pt)

(c) From the result of (a) and (b), determine the lower limit of α required for the existence of a solution for $E < 0$. (3 pts)

4. **(10 points):** Consider a spin-1/2 particle. Note that the Pauli matrices are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Find the eigenvalues and normalized eigenvectors (eigenspinors) of the spin operator $\hat{S}_y$. (3 pts)

(b) A measurement shows that this spin-1/2 system is in an eigenstate corresponding to the larger eigenvalue of the operator $\alpha\hat{S}_y + \beta\hat{S}_z$, where α and β are real constants. What is the eigenvalue and the corresponding normalized eigenspinor? (4 pts)

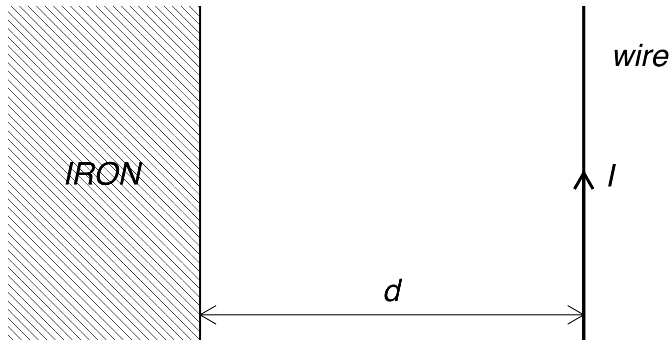
(c) Subsequently, another measurement of $\hat{S}_y$ is carried out. What is the probability that this measurement yields the result of $-\hbar/2$? (3 pts)

5. **(10 points):** An isolated conducting sphere of radius a has a charge $+Q$.

(a) What is the capacitance of the sphere when it is infinitely far from other charges or objects? (3 pts.)

(b) The same sphere is located a distance d from a grounded infinite conducting plane. If $d \gg a$, the capacitance of the sphere can be written as a power series expansion in a/d, and the first term is the value found in (a). Find the next (non-vanishing) term in the power series. (7 pts.)

6. **(10 points):** A long thin wire carrying a current I lies parallel to and at a distance d from a semi-infinite slab of iron, as shown below. Assuming the iron to have infinite permeability, determine the magnitude and direction of the force per unit length on the wire.



Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

FRIDAY, JANUARY 6, 2012
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** Consider a spin-1/2 electron in a hydrogen atom with orbital momentum $l = 1$.

   (a) What are the possible values of the total angular momentum J and projections of the total angular momentum on z axis? (1 pt.)

   (b) For all possible j and j_z derive the corresponding wave functions in terms of spherical harmonics Y_l^m and spinors $\chi_\pm$.

   Now, assume that the electron is in the state described by the following wave function:

$$R(r)\frac{1}{\sqrt{6}}\left(Y_1^0\chi_+ + \sqrt{5}Y_1^1\chi_-\right)$$

   where $R(r)$ represents the (normalized) radial part of the wave function. (5 pts.)

   (c) If one measures the z components of the total angular momentum, what are the possible outcomes? What are the corresponding probabilities? (1 pt.)

   (d) If one measures the total angular momentum, what are the possible outcomes? What are the corresponding probabilities? (3 pts.)

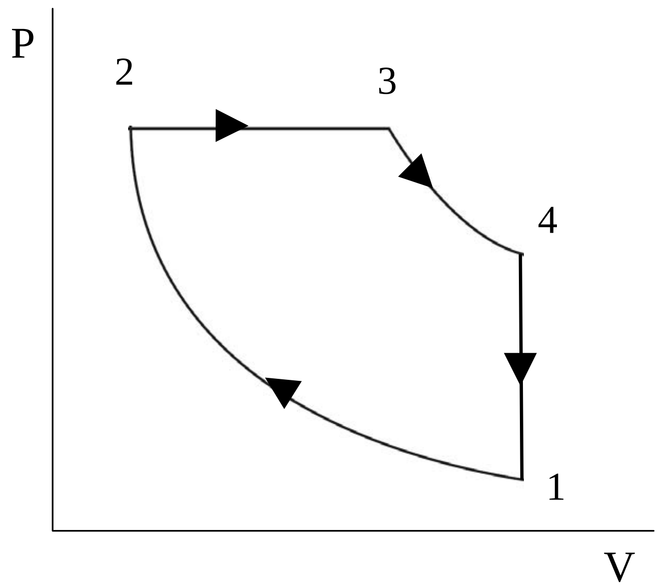
The raising and lowering operators for angular momenta could be useful:

$$J_+ |j, m\rangle = \hbar\sqrt{(j-m)(j+m+1)}\, |j, m+1\rangle$$

and

$$J_- |j, m\rangle = \hbar\sqrt{(j+m)(j-m+1)}\, |j, m-1\rangle\,.$$

2. **(10 points):** Calculate the efficiency of the Diesel cycle shown below, consisting of two adiabats, $1 \rightarrow 2$ and $3 \rightarrow 4$, one isobar, $2 \rightarrow 3$, and one constant volume process, $4 \rightarrow 1$. Assume C_P and C_V for the ideal gas. Express the answer in terms of the compression ratios V_3/V_1 and V_2/V_1.



3. **(10 points):** Two identical bosons (particles 1 and 2) are subjected to the one-dimensional harmonic oscillator potential

$$V(x_1, x_2) = \frac{1}{2}m\omega^2(x_1^2 + x_2^2).$$

(a) For each particle, which is a 1D simple harmonic oscillator, the eigenfunction of the ground state can be written as

$$\psi_0(x) = A\exp(-bx^2).$$

Determine the constants A and b, and find the energy eigenvalue of the ground state. (3 pts)

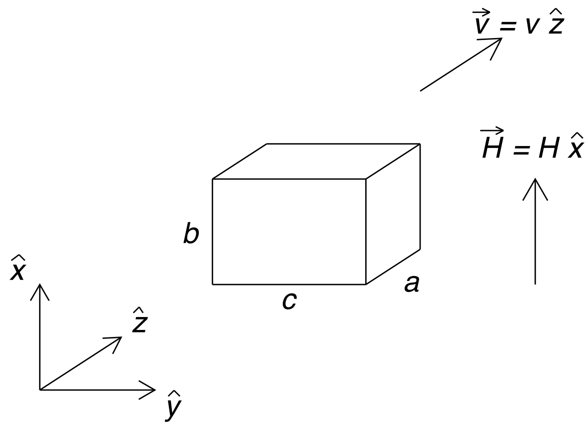
(b) What are the ground-state energy and wave function for this system of two identical bosons? (2 pts)

(c) If these two bosons interact with each other through the potential

$$V'(x_1, x_2) = \beta\exp(-a(x_1 - x_2)^2),$$

where $a > 0$, use perturbation theory to find the first order correction to the ground state energy. (5 pts)

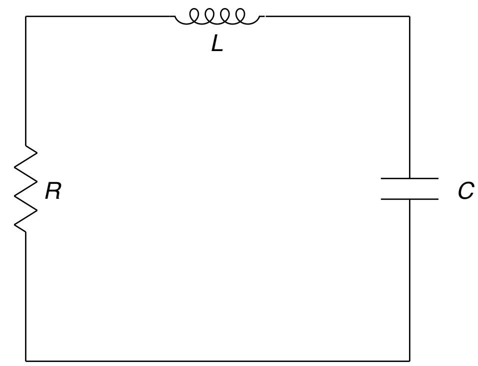
4. **(10 points):** An uncharged, rectangular, metal block has sides of length a, b, and c. It moves with velocity $\mathbf{v} = v\hat{\mathbf{z}}$ in a uniform magnetic field of intensity $\mathbf{H} = H\hat{\mathbf{x}}$. The edges of length a are parallel to the z axis, the edges of length b are parallel to the x axis, and the edges of length c are parallel to the y axis, as shown in the figure.



(a) What is the electric field inside the block? (6 pts.)

(b) What is the electric charge density in the volume and on the surface of the block? (4 pts.)

5. **(10 points):** A circuit consists of a resistor R, a capacitor C, and an inductor L connected in series, as shown below.



(a) Write down the differential equation for the charge flowing in the circuit. (2 pts.)

(b) Let $R = \sqrt{2L/C}$. Solve for charge on the capacitor and current in the circuit when, initially, the capacitor has charge Q_0 and there is no current. (7 pts.)

(c) Does the current oscillate? If it oscillates, at what frequency? If it doesn't oscillate, does the current change its direction of flow? (1 pts.)

6. **(10 points):** A block of mass m is given an initial velocity v_0 up an inclined plane. The coefficient of sliding friction between the block and the plane is $\mu = 0.1$. The inclined plane is fixed in position and has an angle of $\theta = 30°$.

   (a) How far up the plane does the block slide?

   (b) The block slides back down the inclined plane. How much time does it take for the block to slide up and back down to where it started from?