Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART II

MONDAY, MAY 7, 2012
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part II).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **(10 points):** Consider a system of three spins arranged in an equilateral triangle, each spin interacting with the other two. Each spin can only point up or down with the values of $s = \pm 1$, respectively. The energy of the spins in a magnetic field B is described by

$$H = -J(s_1 s_2 + s_2 s_3 + s_3 s_1) - F(s_1 + s_2 + s_3),$$

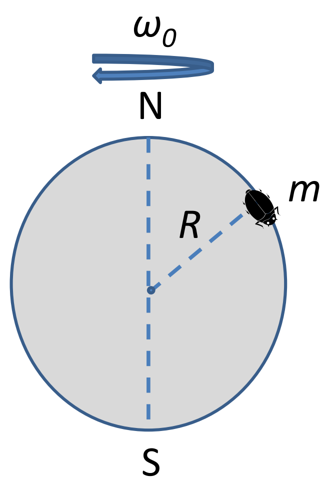
where $F = \mu B$.

   (a) Find the partition function for the system.

   (b) Determine the average spin.

   (c) Calculate the average energy ϵ.

2. **(10 points):** A small globe (a solid sphere of uniform density) rotates without friction with an angular velocity ω_0. A bug starts at the north pole N and travels to the south pole S along a meridian with a constant velocity v. The rotation of the globe is fixed along the north–south axis. The mass and the radius of the globe are M and R respectively, the mass of the bug is m, and the total duration of the bug's journey is T .



(a) Find the moment of inertia of the globe. (1 pt.)

(b) Find the angle of rotation of the globe during the time the bug is traveling. Does the answer make sense in the limit when m goes to zero? (9 pts.)

Hint: The following integral might be useful: $\displaystyle\int_0^{2\pi} \frac{dx}{a + b\cos x} = \frac{2\pi}{\sqrt{a^2 - b^2}}$.
A meridian is a great circle on the sphere, passing through the north and south poles.

3. **(10 points):** Consider a system of two non-interacting, distinguishable spin-1/2 particles.

(a) What are the possible values of total spin S and the z-component of the spin S_z? What are the corresponding normalized eigenstates (spinors)? (3 pts.)

(b) If initially particle 1 is in the eigenstate of $s_z^{(1)} = +\hbar/2$ and particle 2 is in the eigenstate of $s_x^{(2)} = -\hbar/2$, what is the probability of measuring the value of the total spin to be zero in a measurement of this two-particle system? (7 pts.)

The Pauli matrices may be of use:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

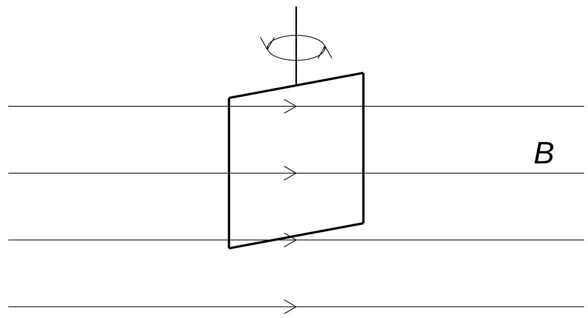
4. **(10 points):** Consider a one-dimensional system with a particle of mass m in an infinitely deep potential well with walls located at $x = -a/2$ and $x = +a/2$. Between the walls the particle is free.

   (a) If the particle is in the ground state, what are the energy and the normalized eigenfunction? (4 pts)

   (b) Suddenly the walls are removed. What is the probability that the momentum of the particle is found to be between p and $p + dp$? (4 pts)

   (c) What is the expectation value of the particle's energy after the walls are removed? (2 pts)

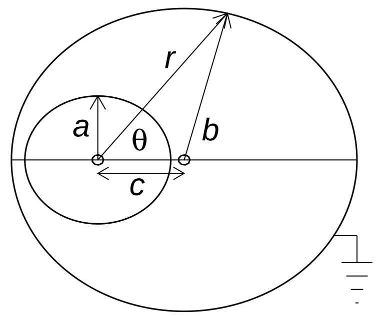
5. **(10 points):** A conducting loop of area A and total resistance R is suspended by a torsion spring of spring constant k in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{y}}$. The loop is in the yz plane at equilibrium and can rotate about the z-axis with moment of inertia I as shown in the figure. The loop is displaced by a small angle θ_0 from equilibrium and released at $t = 0$. Assume the torsion spring is non-conducting and neglect self-inductance of the loop.



(a) What is the equation of motion for the loop in terms of the given parameters? (6 pts.)

(b) What is the motion of the loop at later times in the case that R is large? (4 pts.)

6. **(10 points):** A conducting sphere of radius a is located within a conducting spherical shell of inner radius $b > a$. The inner sphere carries charge Q and the outer shell is grounded. The distance between their centers is c, a distance much smaller than the radius a. NOTE: This distance is exaggerated in the figure for clarity.



(a) Use coordinates with the origin at the center of the inner sphere, and z-axis passing through the center of the shell. Show that the equation describing the radial distance to the outer shell is, to first order in c

$$r(\theta) = b + c\cos\theta$$

where θ is the polar angle indicated in the figure. (2 pts.)

(b) If the potential between the two spheres contains only $\ell = 0$ and $\ell = 1$ angular components, determine the potential to first order in c. Hint: There is axial symmetry in this problem. (8 pts.)