Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, May 3, 2013
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. **10 points** Consider an electromagnetic wave propagating through a linear idealized medium with real dielectric constant ϵ and magnetic permeability μ.

   a) Evaluate the divergence of the Poynting vector as a function of the fields. (3 pts)

   b) Using Maxwell's equations, express the divergence as a function of the rate of change within the medium. (5 pts)

   c) Determine the energy of the travelling wave and the rate of energy transferred to the medium. (2 pts)

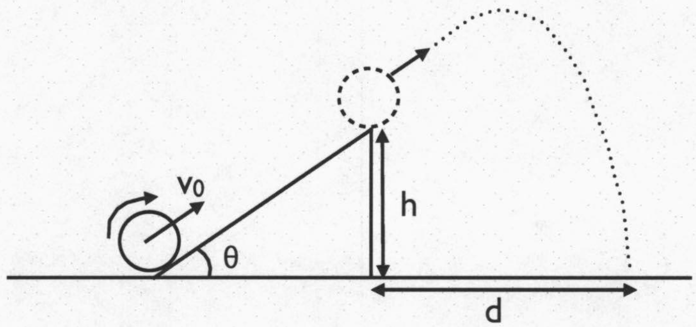
2. **10 points** A uniform spherical ball of radius R, and mass M, rolls (without slipping) up an incline of height h, and angle θ. The ball has an initial velocity v_0 at the bottom of the incline. The velocity v_0 is sufficiently large that the ball projects off the top of the incline and hits the ground a distance d from the end of the incline (see figure).

a) Show that the moment of inertia of the ball in terms of R and M is $I_{ball} = 2/5MR^2$. (2 pts)

b) What is the magnitude and direction of the frictional force as the ball rolls up the incline? (3 pts)

c) What is the magnitude of the ball's velocity as it leaves the incline? (3 pts)

d) What is the distance d from the end of the incline at which the ball hits the ground? (2 pts)



3. **10 points** Consider the three spin one matrices

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

a) Calculate the commutator of S_x and S_y. (2 pts)

b) Compute the values that can be obtained in measuring the spin along the x axis. (4 pts)

c) Suppose the maximum value is obtained when measuring the spin along the $x-$axis. If the spin along the $z-$axis is now measured, what are the probabilities for each possible outcome? (4 pts)

4. **10 points** A monoatomic gas obeys the van-der Waals equation:

$$P = \frac{N\tau}{V - Nb} - \frac{N^2 a}{V^2}$$

where N is the number of particles and a and b are known constants and $\tau = k_B T$. The gas has a heat capacity $C_V = 3N/2$ in the limit $V \to \infty$.

a) Using the thermodynamic identities and the equation of state prove that

$$\left(\frac{\partial C_V}{\partial V}\right)_\tau = 0.$$

(3 pts)

b) Use the result of part a) to determine the entropy of the van-der Waals gas $S(V, \tau)$ to within an additive constant. (4 pts).

c) What is the final temperature when the gas is adiabatically compressed from (V_1, τ_1) to V_2? (3 pts)

5. **10 points** A thin spinning disk of radius R is electrically charged, with a uniform and constant charge surface density σ (consider both sides to be charged). The disk spins with constant angular velocity ω around the axis perpendicular to the plane of the disk.

   a) Calculate the magnetic field at any point along the axis of rotation, $\mathbf{B}(0,0,z)$. (7 pts)

   b) Calculate the total magnetic moment of the spinning disk. (3 pts)

6. **10 points** Consider a quantum mechanical system with two states, $|\alpha>$and $|\beta>$. In this orthonormal basis of states the Hamiltonian is given by the matrix

$$H = \begin{pmatrix} W & V \\ V & -W \end{pmatrix}.$$

a) Obtain the exact energy eigenvalues. (4 pts)

b) Consider the Hamiltonian as $H = H_W + H_V$,

$$H_W = \begin{pmatrix} W & 0 \\ 0 & -W \end{pmatrix}, \quad H_V = \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}.$$

Assuming $W \gg V$, obtain the energy eigenvalues to second order perturbation theory in V. (3 pts)

c) Compare the results obtained in (a) and (b) and verify that they agree to second order in V. (3 pts)

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PART II

MONDAY, May 6, 2013
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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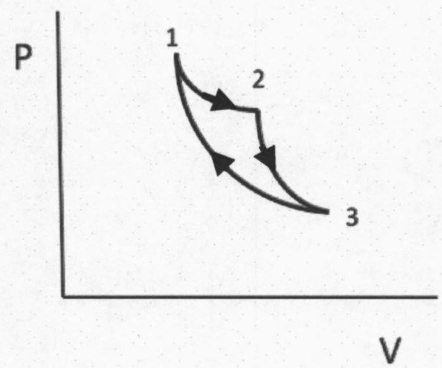
1. **10 points** A gas cycle consists of an isothermal process $1 \to 2$, a polytropic process (a process with a constant heat capacity C) $2 \to 3$, and an adiabatic process $3 \to 1$. See Figure. What is the work done in this cycle? The temperatures at points 1 and 3 are respectively T_1 and T_3.

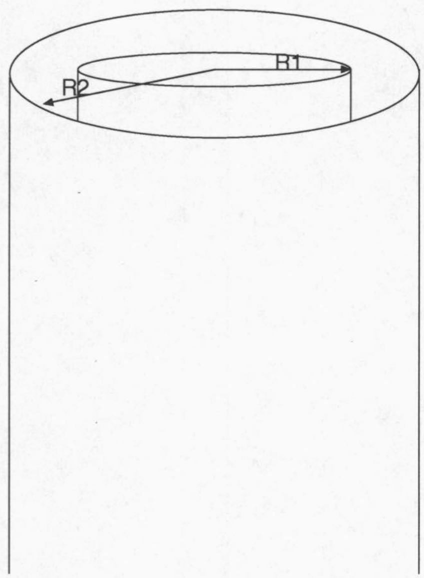


2. **10 points** Consider a body that is confined to move in a vertical plane, the $x - z$ plane. The body has mass m and moves in the plane subject to the (constant) gravitational force g (in the $-\hat{z}$ direction) and an additional "central" force of the form $f = -Ar^{-1/2}$, where $r^2 = x^2 + z^2$. This additional force is thus directed towards the origin. Choose the appropriate generalized coordinates and let the gravitational potential be zero along a horizontal line through the origin ($z = 0$).

a) Find the equations of motion for the system. (7 pts)

b) Show whether or not angular momentum about the origin is conserved. (3 pts)

3. **10 points** Two ideal, very long solenoids of length L have the same axis, see Figure. The number of turns per unit length and radii of the solenoids are $n_{1,2}$ and $R_{1,2}$ respectively, with $R_1 < R_2$. Evaluate the mutual inductance of the two solenoids.



4. **10 points** Consider the electric polarization $\mathbf{P}$ of an ideal gas consisting of N molecules with a constant electric dipole moment p in a homogeneous external electric field $\mathbf{E}$ at temperature T. Ignore any interaction between molecules.

   a) What is the potential energy of a single dipole in an electric field $\mathbf{E}$? (1 pts)

   b) Determine the probability dw that the direction of the dipole is within a solid angle $d\Omega$. (2 pts)

   c) Calculate the total dipole moment of the gas. (5 pts)

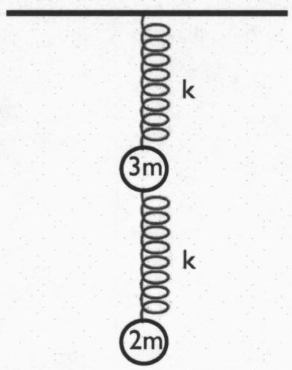
   d) What is the dielectric constant of this gas in the limit of small fields? (2 pts)

5. **10 points** Two ideal massless springs with spring constant k are connected to two masses that hang vertically as shown in the figure. The top one has mass $3m$ and the bottom one has mass $2m$. The system is only able to oscillate in the vertical direction.

a) Determine the equations of motion. (4 pts)

b) Find the frequencies of the normal modes of this system for small vertical displacements. (4 pts)

c) Describe the relative motion and amplitudes of each of the normal modes. (2 pts)



6. **10 points** Consider an electron in a hydrogen atom which has the following wave-function at $t = 0$

$$\psi(0) = A(|100\rangle + 2i\,|210\rangle + 2\,|322\rangle).$$

Here, each of the individual eigenvector terms are denoted by their quantum number N (principal), L (angular momentum), amd M (angular momentum projection) as $|NLM\rangle$.

a) Calculate the value of the normalization constant A. (2 pts)

b) Find the expectation value of the energy of the electron at $t = 0$, and express your answer in eV (hint: the ground level energy of a hydrogen atom is 13.6 eV). (3 pts)

c) If a measurement of the $z-$projection of the electron orbital angular momentum is made at $t = 0$, with what probability are the results $0\hbar, 1\hbar, 2\hbar, 3\hbar$ obtained? (3 pts)

d) Write the expression for $\psi(t)$ at any time $t > 0$. (2 pts)