Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, May 8, 2015
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. Your special ID number that you received from Delores Cowen;
2. The problem number and the title of the exam (*i.e.* Problem 1, Part I).

Please make sure your answers are dark and legible.

Do NOT write your name on the cover or anywhere else in the booklet!

1. (**10 points**) A rocket ship of mass M_0 and loaded with fuel with total mass m starts at rest and takes off vertically in a uniform gravitational field g. The fuel load is ejected uniformly with a velocity u downward with respect to the ship in a total time t_0.

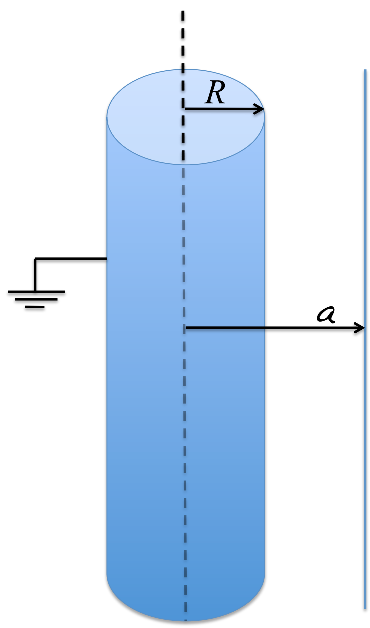
   a)(5 pts) Find the equation of motion of the rocket in terms of the mass of the rocket at time t, M, dM/dt, u, and g.

   b)(5 pts) What is the velocity of the rocket at the instant when all the fuel has been ejected? Give it terms of M_0, m, u, g, and t_0.

2. (**10 points**) An infinite, solid, conducting and grounded cylinder with radius R is placed near and parallel to an infinite charged wire with charge density λ. The wire is at a distance a from the axis of the conducting cylinder.

a)(7 pts) Find the potential and electric field everywhere in this configuration.
HINT: use method of images.

b)(3 pts) Calculate the surface charge density on the grounded cylinder.



3. (**10 points**) A closed vertical cylindrical glass Dewar at room temperature T contains a mixture of two types of gas molecules subjected to Earth's gravity g. N_1 and m_1 are the number and the mass of the type 1 gas molecules. N_2 and m_2 are the number and the mass of the type 2 gas molecules. The container has a cross-section of area a and a height of l. Assuming isothermal condition,

   (a) (3 pts) Calculate the probability of finding a particle at height x ($0 < x < l$) for either type of the gases.

   (b) (3 pts) Find the partition function for the gas mixture.

   (c) (4 pts) Find the pressure on the bottom of the container.

4. (**10 points**) A particle of mass m is confined in a square well potential of the form

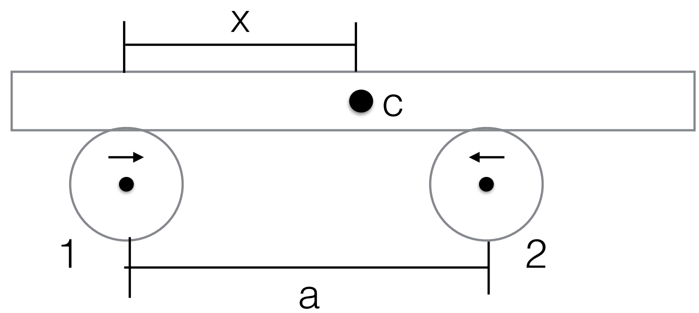
$$U(x) = \begin{cases} -U_0, & |x| < a \\ 0 & |x| > a \end{cases}, \tag{1}$$

such that $ma^2U_0/\hbar^2 \ll 1$.

(a)(5 pts) Prove that there exist only one bound state and find the corresponding energy

(b)(5 pts) Calculate $\langle(\Delta x)^2\rangle$ and $\langle(\Delta p)^2\rangle$

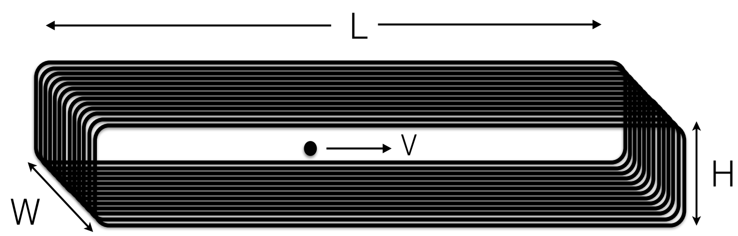
5. (**10 points**) A uniform, thin rigid rod of mass M is supported on two rapidly rotating rollers whose axes are aligned vertically and separated horizontally by a distance a. The rod is initially placed asymmetrically on the rollers as shown in the figure.



a)(7 pts) The rollers are rotating as shown in the figure and there is a coefficient of friction of μ between the rod and the rollers. Give the equation of motion for the rod and solve for the displacement $x(t)$ of the center of the rod C from roller 1 assuming $x(0) = x_0$ and $\dot{x}(0) = 0$.

b)(3 pts) Now consider the case when the direction of rotation of both rollers is reversed. Give the equation of motion for $x(t)$ and qualitatively describe the motion assuming $x(0) = x_0$ and $\dot{x}(0) = 0$.

6. (**10 points**) Consider a solenoid deformed into a cuboid shape as shown in the figure. The dimensions are such that $L \gg W \gg H$. There are $N \gg 1$ turns of the wire such that $n = N/W$ is also large. In the rest frame of the solenoid a current I flows through the solenoid. At $t = 0$ at the midpoint of the solenoid, a charged particle with a small charge q moves with a relativistic velocity V in the x direction, parallel to the length L of the solenoid.



a)(1 pt) Approximating L, W to be very large, use Ampere's law to find the magnetic field within this deformed solenoid. Find the direction and magnitude of the 3-force (Lorentz force) on the particle

b)(3 pts) Boost to the frame where the particle is at rest and the solenoid moves in the negative x direction. Use relativistic transformations of the force to find the 3-force on the particle in this frame.

c)(6 pts) Since the particle is at rest in the new frame, quantitatively explain the origin of the force on the particle in the new frame. Note that since the particle is not moving in its rest frame, there cannot be a magnetic component of the force. Simply boosting the electromagnetic tensor is not sufficient as an answer to this problem, though it may be helpful to you in figuring out the origin of the force in the particle's rest frame.

Ph.D. QUALIFYING EXAMINATION
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PART II

Monday, May 11, 2015
9:00 AM — 1:00 PM

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of six problems, each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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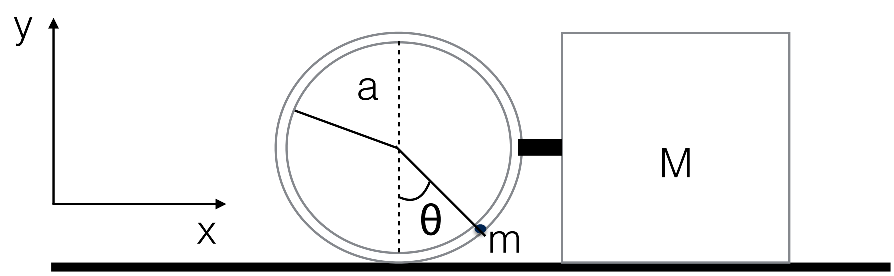
1. (**10 points**) Consider a gas of N particles in volume V in equilibrium with a thermal bath at temperature T.

   (a) (4 pts) Show that the system energy fluctuations can be expressed in terms of the temperature and the system's specific heat (at constant volume).

   (b) (4 pts) Evaluate the energy fluctuations explicitly in the case of an ideal monoatomic gas.

   (c) (2 pts) How the fluctuations change in the case of a diatomic gas?

2. (**10 points**) A block of mass M is rigidly connected to a massless circular track of radius a on a frictionless horizontal table as shown in the figure. A particle of mass m is confined to move without friction on the the vertical circular track.



a)(3 pts) Set up the Lagrangian, using θ as one coordinate.

b)(4 pts) Find the equations of motion.

c)(3 pts) In the limit of small angles, solve the equations of motion for θ and give the angular frequency of the oscillations of m.

3. (**10 points**) Consider a spin $s = 1/2$ particle in a constant uniform magnetic field of strength B_0 pointing along the z-axis.

   (3 pts) Find the eigenfunctions of the spin operator (for all three components)

   (3 pts) Find the eigenfunctions of the Hamiltonian and corresponding energy levels.

   (4 pts) Find the time dependence of the expectation values of spin projections on all three axes if the system is initially in the state with s_y=1/2.

4. (**10 points**) A 1 cm^3 metal cube at room temperature contains 2×10^{22} electrons.

a) (4 pts) Write down an expression for the density of states (DOS) $D(E)$. Evaluate the Fermi energy and thus prove that the system is a degenerate electron gas.

b) (2 pts) Find the Pauli paramagnetic susceptibility χ.

c) (2 pts) Find the pressure against the wall.

d) (2 pts) Find the average kinetic energy $< E >$.

5. Consider a long cylindrical wire of radius R carrying a current I flowing parallel to the direction of the axis of the wire. Within the wire the current density is j(r) , where r is the distance from the wire axis, and the direction is always parallel to the wire axis. This generates a magnetic field which may or may not cause a non-uniform charge density $\rho(r)$ in the wire. The wire is in equilibrium such that $j(r)$ and $\rho(r)$ do not depend on time and the drift velocity of the moving charges is a constant, v_D. Assume $\rho(r) = e\rho_L - e\rho_D$ where ρ_L is the uniform, fixed density of the unmoving, ionized atoms and $\rho_D(r)$ is the density of drift electrons (such that $j(r) = -e\rho_D(r)v_D$).

   a) (6 pts) Solve for $\rho_D(r)$ and $j(r)$. HINT: Use Gauss's Law and Ampere's Law noting the equilibrium state of the system.

   b) (4 pts) What is the stress on the wire due to the charge and current density? The expression for the EM Stress Tensor is

$$T_{ij} = \frac{\epsilon_0}{2}\left[E_i E_j - \frac{E^2\delta_{ij}}{2}\right] + \frac{1}{2\mu_0}\left[B_i B_j - \frac{B^2\delta_{ij}}{2}\right]$$

6. (**10 points**) Consider a charged particle in the ground state of the oscillator potential $U = m\omega^2/2$ under the influence of a weak electric field pointing along the direction of oscillation. Treating the interaction of the charge with electric field as perturbation,

(a)(3 pts) find the corrections to the energy level in the first order of perturbation theory.

(b)(4pts) find the corrections to the energy level in the second orders of perturbation theory.

(c)(3 pts) Compare with the exact solution.

You might find the following equations useful:

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x) \quad (1)$$

$$\hat{a}_{+}|n\rangle = \sqrt{(n+1)}\,|n+1\rangle;$$

$$\hat{a}_{-}|n\rangle = \sqrt{n}\,|n-1\rangle \quad (2)$$