Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I: CLASSICAL MECHANICS

Wednesday, January 6, 2016
10 AM — 12 noon

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the exam and problem number (*i.e.* I.1).

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Problem I.1. A cube of mass M slides without friction down an inclined plane with elevation angle θ. The cube starts its motion from rest and travels a distance s after a time t. A table of integrals follows for use in this problem.

a) (2 pts.) Using conservation of energy, find kinetic energy $K(s)$ and velocity $v(s)$.

b) (2 pts.) Find $s(t)$ by integration, using the result in a) for $v(s)$.

Now consider a solid cylinder of mass m, radius R, and moment of inertia I, rolling without slipping down the same inclined plane. The cylinder also starts from rest and travels a distance s after a time t.

c) (3 pts.) Find $v(s)$ using conservation of energy.

d) (3 pts.) Find $s(t)$ by integration. Show that for $I = 0$ the formula for $s(t)$ reduces to that found in b).

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{1}$$

$$\int \frac{1}{x} dx = \ln|x| \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2}\ln|a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a\tan^{-1}\frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a}\, dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a}\, dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{20}$$

$$\int \sqrt{ax+b}\, dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3}(x \mp 2a)\sqrt{x \pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b}\, dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)}\, dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)}\, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2\ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c}\, dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx^{+}c)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c} \times \left(-3b^2 + 2abx + 8a(c+ax^2)\right) + 3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax\, dx = x\ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2}(\ln ax)^2 \tag{43}$$

$$\int \ln(ax+b)\, dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{44}$$

$$\int \ln(x^2+a^2)\, \mathrm{d}x = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2)\, \mathrm{d}x = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right) dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)\, dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2 - b^2x^2\right) dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) \tag{49}$$

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a}e^{ax} \tag{50}$$

$$\int \sqrt{x}e^{ax} dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \quad \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt \tag{51}$$

$$\int xe^x dx = (x-1)e^x \tag{52}$$

$$\int xe^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{53}$$

$$\int x^2e^x dx = \left(x^2 - 2x + 2\right)e^x \tag{54}$$

$$\int x^2e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{55}$$

$$\int x^3e^x dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{56}$$

$$\int x^ne^{ax}\, \mathrm{d}x = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\, \mathrm{d}x \tag{57}$$

$$\int x^ne^{ax}\, \mathrm{d}x = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\, \mathrm{d}t \tag{58}$$

$$\int e^{ax^2}\, \mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2}\, \mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int xe^{-ax^2}\, \mathrm{d}x = -\frac{1}{2a}e^{-ax^2} \tag{61}$$

$$\int x^2e^{-ax^2}\, \mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

Problem I.2. A thin rectangular plate of mass m and dimensions a, b $(a > b)$ rotates around an axis $\vec{\omega}$ along one of its diagonals (see figure). In the plate's reference frame, side a is along the $x-$axis, side b is along the $y-$axis, and the origin of the coordinate system is at the center of gravity of the plate.

a) (3 pts.) Compute the inertia tensor.

b) (2 pts.) Find the eigenvectors and the eigenvalues of the inertia tensor.

c) (2 pts.) Evaluate the angular momentum in the plate's system.

d) (3 pts.) Evaluate the torque acting on the plate.

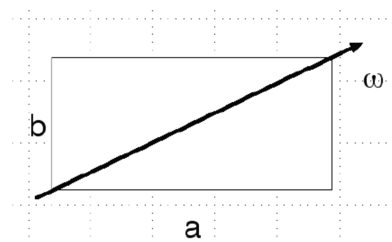

Figure 1: A thin rectangular plate with dimensions a, b $(a > b)$ rotates around an axis $\vec{\omega}$ along one of its diagonals.

Problem I.3. A bead of mass m is constrained to slide on a stiff wire rotating in the $x-y$ plane with angular velocity ω_0. The bead is attached to the pivot point of the wire via a spring of strength k and natural length d.

a) (3 pts.) Find the Lagrangian of the bead and the equations of motion.

b) (3 pts.) Solve the equations of motion and find the positions and velocities as a function of time. Assume that the particle is initially at rest at a distance d from the pivot point (the spring is not stretched).

c) (2 pts.) Evaluate the torque on the particle as a function of position and velocity.

d) (2 pts.) Evaluate the reaction force of the wire on the bead, and the maximum reaction force.

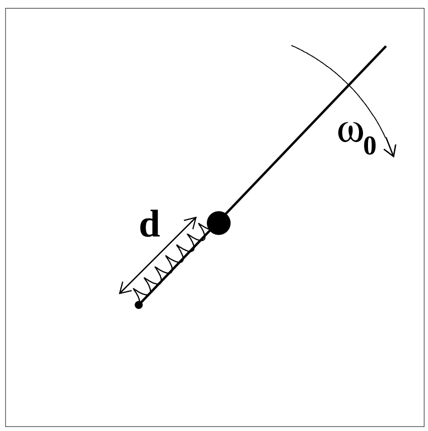

Figure 2: A bead is constrained to slide on a stiff wire rotating with angular velocity ω_0. The bead is attached to the pivot point of the wire via a spring of natural length d.

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PART II: ELECTRICITY AND MAGNETISM

Wednesday, January 6, 2016
1:30 PM — 3:30 PM

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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Problem II.1. A solid insulating charged sphere of radius R has a spherical cavity of radius a cut inside at location $\vec{b}$ relative to the center of the sphere. The remaining sphere has a fixed uniform positive charge density ρ (charges do not move within the sphere). The sphere is located near an infinite conducting grounded plate, with its center a distance d from the plate.

a) (1 pt.) What is the total charge on the sphere?

b) (2 pts.) What is the electric field in the vicinity of the sphere (both inside and outside the sphere) if the conducting plate is not present?

c) (3 pts.) What is the electric field in the entire hemisphere containing the sphere, above the grounded plane?

d) (3 pts.) What is the induced charge density on the conducting plate?

e) (1 pt.) What is the total induced charge on the conducting plate?

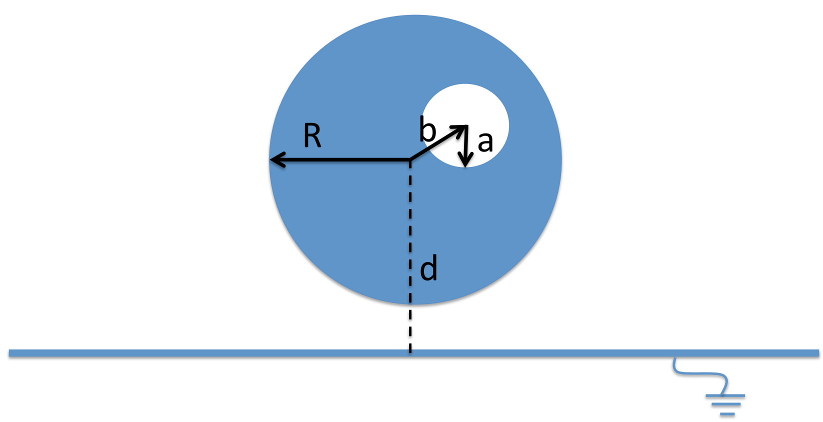

Figure 1: A solid insulating charged sphere of radius R has a spherical cavity of radius a cut inside at location $\vec{b}$ relative to the center of the sphere. The sphere is located near an infinite conducting grounded plate, with its center a distance d from the plate.

Problem II.2. An air core (cylindrical) solenoid has diameter D and is wound with a wire of length l, diameter d, and resistivity ρ. The solenoid is wound with no space between turns, as shown in the figure. Find the following quantities in terms of the given variables and fundamental constants:

a) (2 pts.) the length L of the solenoid,

b) (1 pt.) the resistance of the wire,

c) (2 pts.) the inductance.

d) (3 pts.) The solenoid is connected to an AC source of frequency f and amplitude V_0. The phase of the voltage is such that $V(t = 0) = 0$. Compute the time dependence of the voltage across the solenoid.

e) (2 pts.) Evaluate I_{rms}.

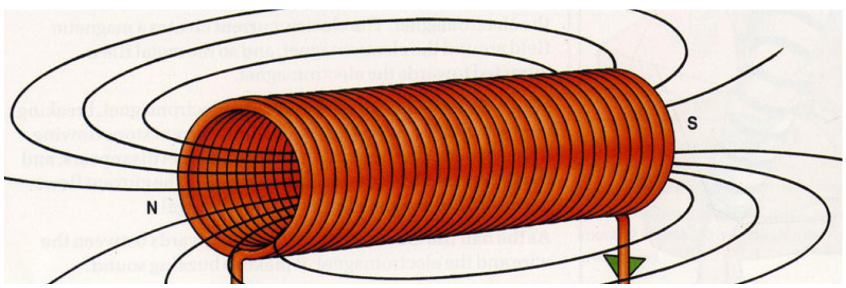

Figure 2: An air core (cylindrical) solenoid is wound with a wire such that there is no space between turns.

Problem II.3. A sphere of radius a is conducting and held at voltage V_0. A second larger spherical thin shell, of radius b, is concentric to the first sphere and insulating. A static charge is present on the insulating shell with density

$$\sigma(\theta) = \sigma_0 \cos\theta = \sigma_0 P_1(\cos\theta),$$

where P_1 is the Legendre polynomial of first order. Recall that the general solution to spherical problems is

$$V(r) = \Sigma_l (A_l r^l + \frac{B_l}{r^{l+1}}) P_l(\cos\theta).$$

a) (2 pts.) Find the potential inside the conducting sphere.

b) (3 pts.) State all boundary conditions.

c) (2 pts.) Write the general solution for the potential outside the sphere ($r > a$) using separation of variables.

d) (3 pts.) Calculate explicitly the potential for $r > a$.

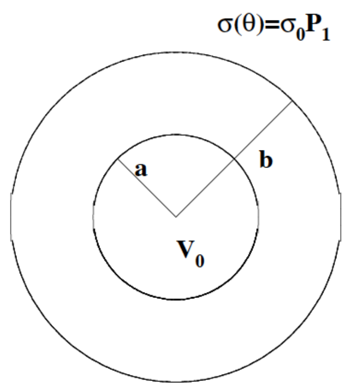

Figure 3: A sphere of radius a is conducting and held at voltage V_0. A second larger spherical thin shell of radius b is concentric to the first sphere and insulating. A static charge is present on the insulating shell with density $\sigma(\theta) = \sigma_0 P_1(\cos\theta)$.

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PART III: QUANTUM MECHANICS

Friday, January 8, 2016
10 AM — 12 noon

Room 245, Physics Research Building

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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Problem III.1. A beam of particles can be described by a quantum-mechanical wave. Consider the 1-dimensional motion of a beam of particles of mass m and energy $E > 0$ traveling (non-relativistically) in the $-x$ direction and incident on a step potential at $x = 0$. (The particles have $x > 0$ before they reach the step.) The potential energy is described by

$$U(x) = \begin{cases} -U_0 & x < 0 \\ 0 & x \geq 0 \end{cases}$$

where U_0 is a positive real number.

a) (2 pts.) State the time-independent Schroedinger equation for each region of x.

b) (2 pts.) State the form of the solutions for each region of x.

c) (2 pts.) State the boundary conditions.

d) (3 pts.) Find the probability that a particle is back-scattered (reflected).

e) (1 pt.) Using the result in d) find the probability for back scattering in the classical limit.

Problem III.2. A one-dimensional quantum harmonic oscillator of mass m has ground state time-independent wavefunction $\psi_0(x)$ for potential energy $V(x) = \frac{1}{2}kx^2$ $(k > 0)$.

a) (6 pts.) Find $\psi_0(x)$ with the correct normalization factor using the lowering operator $\hat{a}$ for which $\hat{a}\psi_n(x) = \sqrt{n}\psi_{n-1}(x)$, where $\hat{a} = i\hat{p} + m\omega x$, $\omega = \sqrt{k/m}$, and $\hat{p}$ is the momentum operator. A mathematical table follows.

b) (4 pts.) At a certain point in time, k is instantly doubled so that the potential enmergy becomes $V_0' = kx^2$ for which the ground state is $\psi_0'(x)$. For the instant when V_0 is changed into V_0', find the numerical value of the probability of finding the particle in state ψ_0'.

Mathematical Formulas

Trigonometry:

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Integrals:

$$\int x \sin(ax)\, dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax)$$

$$\int x \cos(ax)\, dx = \frac{1}{a^2}\cos(ax) + \frac{x}{a}\sin(ax)$$

Exponential integrals:

$$\int_0^\infty x^n e^{-x/a}\, dx = n!\, a^{n+1}$$

Gaussian integrals:

$$\int_0^\infty x^{2n} e^{-x^2/a^2}\, dx = \sqrt{\pi}\,\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$$

$$\int_0^\infty x^{2n+1} e^{-x^2/a^2}\, dx = \frac{n!}{2} a^{2n+2}$$

Integration by parts:

$$\int_a^b f\frac{dg}{dx}\, dx = -\int_a^b \frac{df}{dx} g\, dx + fg\Big|_a^b$$

Problem III.3. A spin 1/2 particle interacts with a magnetic field $\vec{B} = B_0\hat{z}$ through the Pauli interaction $H = \mu\vec{\sigma} \cdot \vec{B}$ where μ is the magnetic moment.

The Pauli spin matrices are $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ where the σ_i are

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The eigenstates for σ_z are the spinors

$$\alpha = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \beta = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

(a) (3 pts.) Suppose that at time $t = 0$ the particle is in an eigenstate χ_x corresponding to spin pointing along the positive x-axis. Find the eigenstate χ_x in terms of α and β.

(b) (7 pts.) For a later time t, find the probability that the particle is in an eigenstate corresponding to the spin pointing along the negative y-axis.

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PART IV: THERMAL, STATISTICAL, AND MODERN PHYSICS

Friday, January 8, 2016
1:30 PM — 3:30 PM

Room 245, Physics Research Building

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Problem IV.1. One mole of a van-der-Waals gas (at pressure P_i and temperature T_i) is held within a container with a movable piston. The walls of the container are adiabatic (cannot absorb any heat). A piston can be used to change the volume V of the gas in the container. There is a valve on the piston that when opened allows gas to freely flow through it. Initially the gas is only in the bottom portion of the container with a volume $V_0/2$.

a) (5 pts.) The valve is opened and the gas is allowed to expand into the remainder of the container (the full volume V_0). Assuming that the specific heat at constant volume c_V is independent of temperature, what is the new temperature of the gas?

b) (5 pts.) Then the piston is drawn completely to the top. The valve is now shut and the piston is pushed down until the entire gas is in the bottom of the container within volume $V_0/2$. What is the new temperature of the gas ?

The van-der-Waals gas has an equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \tag{1}$$

where a, b are small constants.

Maxwell's relation is

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T . \tag{2}$$

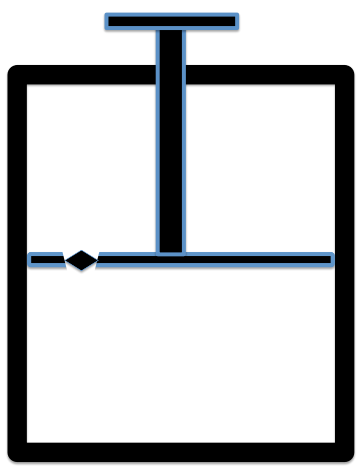

Figure 1: A gas is contained in a volume with a movable piston and a valve.

Problem IV.2. A box contains 4 distinguishable (classical) particles and 4 energy levels $0, a, 2a$ and $3a$ (where a is a positive constant). Ignore zero point energy in all calculations below.

a) (3 pts.) Initially the box is completely thermally insulated, with total energy $4a$ inside. What is the degeneracy of this state ?

b) (3 pts.) In the next scenario, the box is held in thermal contact with a heat reservoir at a temperature $T = 1/\beta$ (Canonical Ensemble). What is the partition function for the box? What is the mean energy at a temperature T of the reservoir? Find the temperature T_{4a} at which the mean energy of the box is equal to $4a$. Just set up the equation for this temperature, but do not solve it.

c) (4 pts.) Now, consider the case where the box in part a) is brought in contact with a reservoir at a temperature of T_{4a}. What is the mean energy transferred between the reservoir and the box? What is the variance (fluctuation) of this energy transfer?

Problem IV.3. A photon of energy E collides with a stationary electron whose rest mass energy is 511 keV. After the collsion the photon scatters at an angle θ (with respect to its original direction) with energy E'. The motion of the electron can be described classically (non-relativistically).

Answer the following questions in terms of the given variables and fundamental constants.

a) (1 pt.) What is the initial wavelength of the photon?

b) (2 pts.) What is the final-state (DeBroglie) wavelength of the electron?

c) (2 pts.) Suppose that the photon loses 100 eV of energy in the collision. Find the factor $\beta = v/c$ of the final state electron.

d) (5 pts.) Denote the ratio of E to electron rest mass energy by r and the ratio of E' to E by f. Find the scattering angle of the *electron* relative to the initial photon direction.