Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

FRIDAY, May 5, 2017
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number(*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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1. **10 points**

   A smooth wire is bent into the shape of a helix, with cylindrical polar coordinates $\rho = R$ and $z = \lambda\phi$, where R and λ are constants and the z-axis is vertically up (and gravity vertically down).

   (5 pt.) Using z as your generalized coordinate, write down the Lagrangian for a bead of mass m threaded on the wire.

   (4 pt.) Find the Lagrange equation and hence the bead's vertical acceleration $\ddot{z}$.

   (1 pt.) In the limit that $R \to 0$, what is $\ddot{z}$? Does this make sense?

2. **10 points**
(4 pt.) By examining the effective potential energy $U_{eff} = -Gm_1m_2/r + l^2/(2\mu r^2)$ find the radius at which a planet (or comet) with angular momentum l can orbit the sun in a circular orbit with fixed radius.
(6 pt) Show that this circular orbit is stable, in the sense that a small radial nudge will cause only small radial oscillations.

3. **10 points**
A railroad car can move on a frictionless track. The railroad car has mass M and is initially at rest. In addition, N people (each mass m) are initially standing at rest on the car.

(3 pt.) Consider the case where all N people run to the end of the railroad car in unison and reach a speed, relative to the car, of V_r. At that point they all jump off at once. Calculate the velocity of the car relative to the ground, after all the people have jumped off.

(6 pt.) Now consider a different case, in which people jump off one at a time with relative speed V_r, while the remaining people remain at rest relative to the car. That continues until all N people have jumped off. Find an expression for the final velocity of the railroad car relative to the ground.

(1 pt.) In which case does the railroad car attain a greater velocity?

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PART II

FRIDAY, May 5, 2017
13:30 — 15:30

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4. **10 points** A static electric field is described by

$$\mathbf{E} = \frac{V_0}{R} \exp\left(-r/R\right)\hat{\mathbf{r}}.$$

(2pt.) Determine the charge density $\rho(\mathbf{r})$.

(2 pts.) Determine the total charge Q.

(2 pts.) Determine the potential $V(\mathbf{r})$.

(2 pts.) Determine the electrostatic energy.

(2 pts.) A small test charge $+q$ is released at $r = R \ln 2$. Find its kinetic energy at infinity.

5. **10 points** A plane wave propagates in vacuum and is described by the equation

$$\mathbf{E}(\mathbf{r}, t) = \frac{V_0}{a} \cos\left(3x/a - 4y/a - \omega t\right)\hat{\mathbf{z}}.$$

In the following, give your answers as a function of V_0, a and electromagnetic constants.

(2pts.) Find the frequency ω and the period T of the wave.

(2 pts.) Find the wavelength λ of the wave.

(3pts.) Derive an equation for the magnetic field $\mathbf{B}$.

(3pts.) Derive an equation for $\nabla \times \mathbf{E}$.

6. **10 points**. A magnetic field $\vec{B}$ is produced from a current i flowing in a circular wire of radius R. The field $\vec{B}$ can be determined from the Biot-Savart law which states that $\vec{B}$ at a point $\vec{r}$ from a current element $id\vec{s}$ is proportional to $id\vec{s} \times \hat{r}/r^2$ with proportionality constant $\mu_0/(4\pi)$ in S.I. units.

   (4 pts.) Find $\vec{B}$ at a point z along the z-axis for $-\infty < z < +\infty$ where the z-axis is perpendicular to the plane of the circular wire and passes through its center.

   (1 pt.) In the limit $|z| >> R$ find the dependence of $\vec{B}$ on z for points along the z-axis.

   (4 pts.) Now consider an electric dipole consisting of a positive charge q located at $z = b$ and a negative charge $-q$ at $z = -b$ along the z-axis. Find the electric field $\vec{E}$ at a point z along the z-axis for $-\infty < z < +\infty$.

   (1 pt.) In the limit $|z| >> b$ find the dependence of $\vec{E}$ on z for points along the z-axis.

   Point of interest: In the far field approximation $|\vec{r}| >> R$ and $|\vec{r}| >> b$ the $\vec{B}$-field of a magnetic dipole and the $\vec{E}$-field of an electric dipole are identical to within a constant of proportionality.

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PART III

MONDAY, May 8, 2017
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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***Problem* 7**

Consider a hydrogen-like (or, hydrogenic) ion U^{91+} of uranium in which all but one of the electrons have been stripped. Assume the ion is at rest in the laboratory frame and is in its ground electronic state.

(a) [4 points] Treating this problem non-relativistically, determine the lowest energy (in eV) of a photon that can be absorbed by this ion.

(b) [5 points] Non-relativistically, the $1s$ hydrogenic orbital for nuclear charge Z is given by

$$\psi_{1s} = \sqrt{\frac{Z^3}{\pi a_o^3}} \exp\left(-\frac{Zr}{a_o}\right)$$

where $a_o = \hbar^2/(me^2)$ is the Bohr radius. Calculate the root-mean-square value of the speed (v_{rms}) of $1s$ electron in hydrogen-like uranium ion by first finding the expectation value of $\langle p \rangle^2$.

(c) [1 point] Compare your result from part (b) with the speed of light. Based on this comparison, comment on the accuracy of photon energy determined in part (a).

[USEFUL INFORMATION: For a function f of scalar r, we have

$$\nabla^2 f(r) = \frac{d^2 f(r)}{dr^2} + \frac{2}{r}\frac{df(r)}{dr}$$]

***Problem* 8**

A particle of mass m is confined inside a one-dimensional quantum well with infinitely high walls at $x = L$ and $x = -L$.

(a) [5 points] Determine the energy levels and the corresponding normalized wave functions of the particle.

(b) [5 points] Now, a weak potential of the form $H'(x) = A\,\delta(x)$ is added as a perturbation. Determine the shift in the energy levels to first order in A.

Problem **9**

A particle of mass m and energy E moves in the potential

$$V(x) = \begin{cases} -V_0 & for\ x < 0 \\ +V_0 & for\ x > 0 \end{cases}$$

where V_0 is a real, positive constant and $-V_0 < E < V_0$.

(a) [5 points] Find solutions $\Psi(x,t)$ to the one-dimensional Schrodinger equation in the region $x < 0$ and $x > 0$. State the boundary conditions and use them to reduce the number of unknown constants.
Now consider that the particle is incident from the left (x<0), moving towards x=0.

(b) [1 point] Find the probability that the particle will be backscattered (reflected).

(c) [4 points] Find an expression for $|\Psi(x, t = 0)|^2$ for the region $x < 0$.

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PART IV

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10. **10 points** A system has three energy levels $\epsilon_i = k_B T_i$ $(i = 1, 2, 3)$, with $k_B = 1.38\ 10^{-23} J/K$ the Boltzman constant. The degeneracies of these energy levels are $1, 2, 4$ respectively. If $T_1 = 0\ K$, $T_2 = 200\ K$ and $T_3 = 400\ K$, then

(3 pt.) calculate the partition function of the system at a temperature of 400 K,
(2 pt.) calculate the relative population of the energy levels at a temperature of 400 K,
(2 pt.) calculate the average energy of the system at a temperature of 400 K.
(3 pt.) At what temperature is the population of the energy level ϵ_3 equal to the population of the energy level ϵ_2?

11. **10 points** A cylindrical vessel of length D is separated into two compartments by a thin sliding, thermally conductive partition, originally held at D/3 from the left end. The left side is filled with $n_L = 1$ moles of an ideal monoatomic gas with pressure $P_L > 1$ atmospheres, the right side contains an unknown quantity n_R of another ideal monoatomic gas at $P_R = 1$ atmosphere of pressure. The entire system is thermally isolated from the surroundings and initially at thermal equilibrium at temperature T.

    The clamp holding the piston is now removed, letting the piston slide to the right. After a long time,

    (1pt.) what are the temperature changes in each partition?

    (4pt.) how far from the left end will the piston go?

    (5 pts.) Starting from $dS = (\frac{\partial S}{\partial V})_T dV + (\frac{\partial S}{\partial T})_V dT$, compute the change in entropy of the system.

12. **10 points**. In the lab frame a photon of energy E_1 moves along the $+z$ direction towards a photon of energy E_2 ($E_2 < E_1$) that moves along the $-z$ direction. The two photons collide and scatter.

(1 pt.) What is the direction of the center-of-mass (CM) frame?

(3 pts.) Find the momentum of each photon in the CM frame that moves with speed v, $v > 0$.

(2 pts.) Find v.

In the CM frame, the photons scatter at an angle of 90 degrees with respect to the initial line of collision.

(4 pts.) Find the photon energies E_A and E_B in the lab frame after the scattering.

Note of interest: The cross-section for scattering of optical photons is tiny. However, gamma-ray energy photons can have a substantial scattering cross section. Classical electromagnetic theory cannot describe light-light scattering; quantum mechanics and special relativity are required.