Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, Jan. 4, 2019
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number (*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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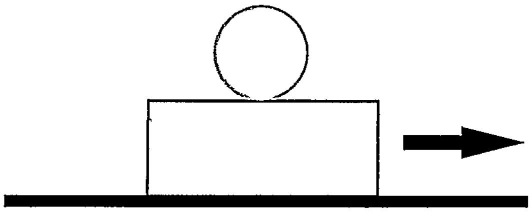
-1- (10 points) A disk of radius R is on a slab. At t=0, a constant force F is applied to the slab (see Figure). Both the disk and the slab have mass M. The slab-support interface is frictionless, but the disk does not slip.



(a) (1pt) Compute the moment of inertia of the disk.
(b) (3 pts) Write equations for the forces acting on the slab and the disk
(c) (2 pts) Write an equation for the torque acting on the disk.
(d) (2 pts) Evaluate the linear accelerations of both the disk and the lab.
(e) (2 pts) Evaluate the minimum coefficient of friction required for the disk to roll without slipping.

-2- (10 points) Consider two identical atomic clocks of very high precision. The first is located at the center of the Earth. The second is on board a satellite in a circular orbit around the Earth (above the equator) at an altitude of 300 km.

(a) (4 points) Calculate the speed of the satellite on its orbit and its orbital period (as measured from Earth)

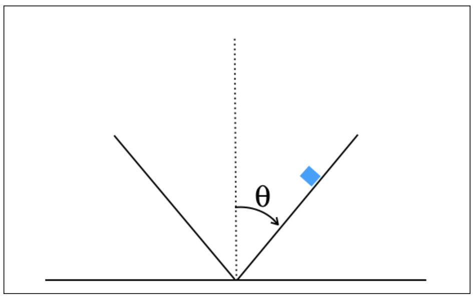
(b) (2 point) Does the clock on the satellite run faster or slower than one on the ground? Quantify the difference.

(c) (4 point) What is the time difference accumulated each time the satellite completes an orbit. Ignore the effect of Earth's rotation or GR effects.

Treat the Earth as a perfect sphere of radius 6370 km and mass 5.98×10^{24} kg and $G=6.67408 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

-3- (10 points) Consider a particle of mass m constrained to move on the inside surface of a cone with angle θ. The axis of the cone is parallel to the axis z as shown in the figure. The surface of the cone is frictionless and there is a uniform gravitation field $\vec{g} = -g\hat{z}$.

(a) (3 points) Determine the Lagrangian of the particle.
(b) (2 points) Determine the constants of motion.
(c) (2 points) Show that a particular solution consisting of a circular orbit of radius $r=r_0$, where r is the distance to the z axis. Discuss the condition(s) under which such orbit happens.
(d) (3 points) Show that this orbit is stable w.r.t. to small perturbations and find the frequency of small oscillations of r around r_0.



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PART II

FRIDAY, Jan. 7, 2019
13:30 — 15:30

ROOM 245 PHYSICS RESEARCH BUILDING

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4. **(10 points)** A RLC circuit is composed of the three elements R,L,C, and is driven with a AC source of maximum amplitude (peak potential) V_0 and angular frequency ω.

   a) (1pt.) Determine the impedance Z and resonant frequency ω_0.

   b) (3pts.) Determine the peak charge value Q_0 on the capacitor as a function of the given variables.

   c) (3pts.) Determine the time delay (positive or negative) between the source getting to V_0 and the capacitor getting to maximum charge.

   d) (3 pts.) Determine the value of ω for which Q_0 is maximal.

5. **(10 points)** An infinite grounded conducting plane is at $z = 0$. A charged wire with linear charge density λ is parallel to the plane and is described by $z = d, x = 0$.

a) (5pt.) Compute the potential V in the region $z > 0$.

b) (5pts.) Determine the capacitance per unit length for the configuration, given the radius of the wire $a \ll d$.

6. **(10 points)** A thin rod of length L has linear charge density λ. It is laying in the $(x - y)$ plane and rotated with constant angular velocity ω about one of its ends, with the axis of rotation along the $z-$axis.

a) (3pt.) compute the electric dipole $\mathbf{p}$ as a function of time.

b) (3pts.) compute the magnetic dipole $\mathbf{m}$ as a function of time.

c) (4pts.) compute the average power emitted by the rotating rod.

Oscillating Dipole power formulae as a function of time:

$$W_e(t) = \frac{\dot{p}^2\omega^2}{6\pi\epsilon_0 c^3}, W_m(t) = \frac{\dot{m}^2\omega^2\mu_0}{6\pi c^3}$$

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PART III

MONDAY, Jan. 7, 2019
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

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7. **(10 points)** A particle rotates at fixed radius subject to the Hamiltoinan

$$H = A(L_x^2 + L_y^2) + BL_z^2,$$

where A and B are constant parameters and $\vec{L}$ is the orbital angular momentum.

a) (3 pts.) State the energy eigenvalues if $A = B$ in terms of the relevant angular momentum quantum numbers. What is the degeneracy of each energy level? Sketch the energy levels of the lowest four states.

b) (4pts.) Find the energy eigenvalues for $A \neq B$. Sketch the energy levels of the lowest four states assuming $A > B$. Are any states still degenerate?

c) (3 pts.) Find $\langle L_x \rangle$, $\langle L_x^2 \rangle$, and the variance of L_x for these states. Hint: be sure to exploit all the symmetries of the system.

8. **(10 points)** A particle of mass m moves in a uniform potential of constant depth $-V_0$.

a) (2 pts.) State the Hamiltonian of the system and obtain the time dependent Schrödinger equation.

b) (4pts.) Solve the Schrödinger equation and find the wave functions and energy eigenvalues E.

c) (2pts.) Find the average momentum and average velocity of each of these states.

d) (2pts.) Find the phase and group velocities corresponding to these wave functions. Compare your answers to $\langle v \rangle$ from part c, Can you explain the physical significance of any differences or similarities?

9. **(10 points)** An electron is confined in a 1-D infinite quantum well

$$V(x) = 0 \text{ if } |x| < a, \; \infty \; \text{ if} |x| > a.$$

The wave function at $t = 0$ is

$$\psi(x, 0) = C(3 \sin \pi x/a + 4 \cos 3\pi x/a).$$

a) (2pts.) Determine the wavefunction and energy eigenvalues for the three lowest energy states.

b) (1 pts.) Determine the normalization constant C.

c) (2pts.) Determine the state at $t = 0$ as a function of the lower states you just found, and its average energy.

d) (2 pts.) Write the wavefunction for arbitrary time t.

e) (3 pts.) Find the average position as a function of time for arbitrary time t.

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PART IV

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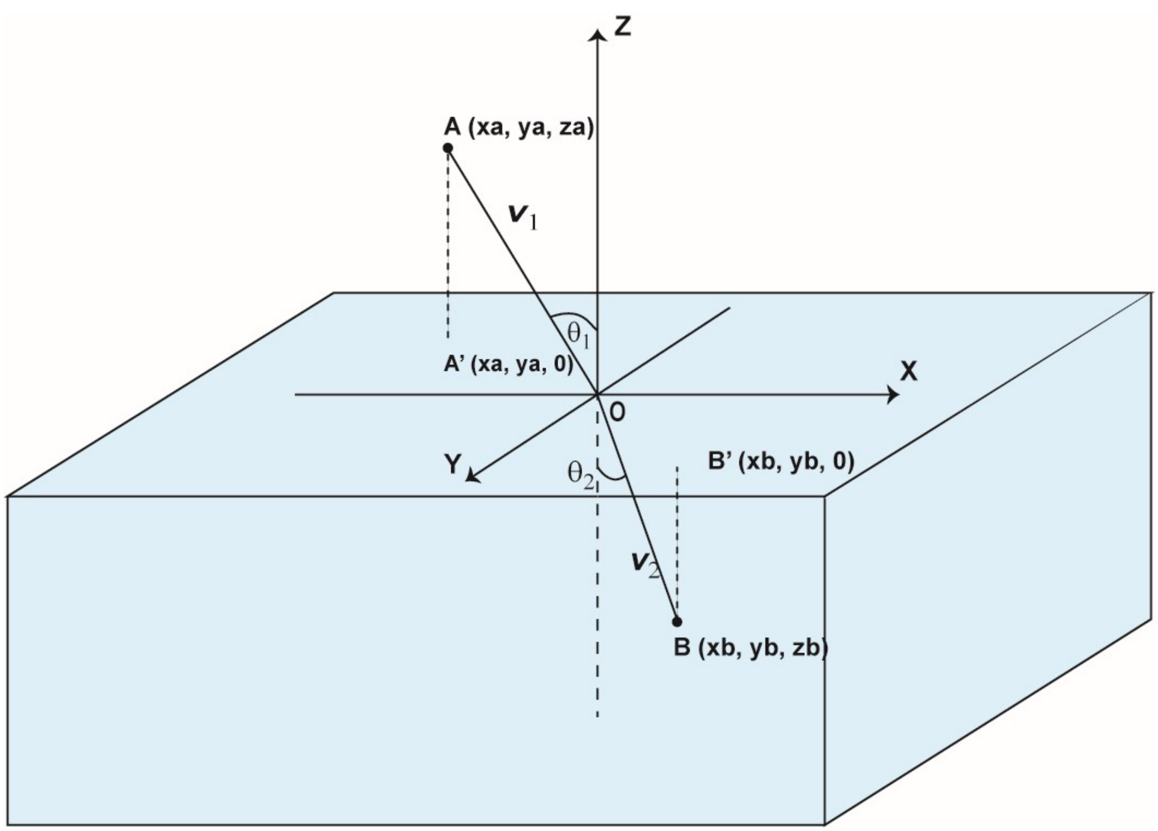
***Problem* 10 (10 pts)**

Fermat's principle states that the path taken between two points by a ray of light is the path that can be traversed in the least time. Light propagates from point A (xa,ya,za) in a medium with an index of refraction n_1 to point B (xb,yb,zb) in a medium with an index of refraction n_2 through a planar interface. Assume that in the figure, the points A and B have equal distance h from the plane (in the figure, AA'=BB'). The velocities of the light in the medium n_1 and n_2 are v_1 and v_2, respectively. The angle of incidence from A to O and the angle of refraction are θ_1 and θ_2.

(a) [5 points] derive the time it takes for light to propagate from A to a point O on the interface, and from O to B. The plane containing A, O and B is perpendicular to the interface.

(b) [5 points] Use Fermat's principle to derive the relationship between θ_1 and θ_2 in terms of the respective velocities v_1 and v_2 in the figure.



Problem **11 (10 pts)**

Consider an ideal gas of N particles of mass m confined in a cubic box with side L. The inside of the box is at equilibrium temperature T and is in a uniform gravitational field with gravity constant g.

(a) (3pts.) Find the density $\rho(z)$ of the gas as a function of vertical position in the container where the box placed in near the earth surface. Note that the gravitational force acts on the particles as g, and the potential energy of the particle is $U(z) = mgz$ where z is the vertical coordinate inside the box.

(b) (4pts.) Find the entropy of the gas as a function of N, V, and g in the limit of $mgL / k_B T << 1$. Note that the canonical partition function can be written:

$$Z = \frac{1}{N!}\left(\left[\frac{2\pi m\ {}_B T}{h^2}\right]^{3/2} L^3 \int_0^L \frac{dz}{L} e^{-\beta mgz}\right)^N$$

(c) (3pts.) The container is sent to a region of space without gravity. In this situation, you can consider that the temperature is constant and the same throughout the container. Find the change in entropy relative to problem (b).

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Problem **12 (10 pts)**

Consider a fixed number of particles with mass m. Consider the free energy F as a function of the internal energy U, the temperature, and the entropy S:
F = U – TS

(a) (2 pts.)Write an expression for $(\frac{\partial S}{\partial V})_T$ in terms of temperature, pressure, and volume.

(b) (4 pts.) Prove the equation below by using the result of (a)

$$(\frac{\partial U}{\partial T})_V = -T(\frac{\partial P}{\partial T})_V(\frac{\partial V}{\partial T})_S$$

(c) (4pts.) Show that

$$(\frac{\partial T}{\partial V})_S = \frac{(\frac{\partial V}{\partial T})_P(\frac{\partial P}{\partial V})_T}{(\frac{\partial S}{\partial T})_V}$$