Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, January 3, 2020
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number (*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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Problem 1

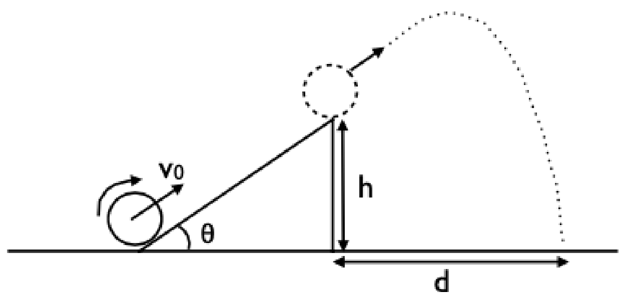
A uniform spherical ball of radius R, and mass M, rolls (without slipping) up an incline of height h, and angle θ. The ball has an initial velocity v_0 at the bottom of the incline. The velocity v_0 is sufficiently large that the ball projects off the top of the incline and hits the ground a distance d from the end of the incline (see figure).

(a) Show that the moment of inertia of the ball in terms of R and M is $I_{\text{ball}} = \frac{2}{5}MR^2$. [2 points]

(b) What is the magnitude and direction of the frictional force as the ball rolls up the incline? [3 points]

(c) What is the magnitude of the ball's velocity as it leaves the incline? [3 points]

(d) What is the distance d from the end of the incline at which the ball hits the ground? [2 points]



Problem 2

A particle is subject to a central force $F(r) = -k/r^\alpha$, k being a given constant.

(a) Prove that the orbit must be circular if the particle energy is equal to its equivalent potential energy $V(r) = U(r) + l^2/(2\mu r^2)$, were l is the particle angular momentum and μ the reduced mass. Find the value of the radius r_0. [3 points]

(b) What is the range of values of α for which the circular orbit is stable? [4 points]

(c) Within the α interval that you found, compute the frequency of small radial oscillations around the nominal circular orbit, when the particle is perturbed by a small radial perturbation. [3 points]

Problem 3

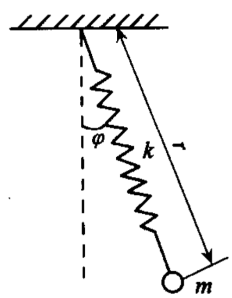
Consider a ball with mass m connected with a spring to the ceiling. The spring can be considered as massless. It has a natural length l_0 and a spring constant k. The system is constrained in a 2D vertical plane.

(a) Compute the length of the spring at equilibrium, r_0. [1 point]

(b) Write out the Lagrangian with the general coordinates, $\lambda = \frac{r-r_0}{r_0}$ and ϕ. [2 points]

(c) Derive the Lagrangian equations with the following definition $\omega_s^2 = \frac{k}{m}$ and $\omega_p^2 = \frac{g}{r_0}$. [4 points]

(d) Solve the Lagrangian equations with the approximation that λ and ϕ are small, only keeping the linear order terms in $\lambda, \phi, \dot{\lambda}, \dot{\phi}, \ddot{\lambda}, \ddot{\phi}$, and neglecting $\mathcal{O}(\lambda^2, \phi^2, \dot{\phi}^2, \dot{\lambda}\dot{\phi})$ and higher. The initial conditions at $t = 0$ is specified as follows, $\phi(t = 0) = 0$, $\lambda(t = 0) = A$, $\dot{\phi} = \omega_p B$, $\dot{\lambda} = 0$. Here A and B are constants. [3 points]



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PART II

Friday, January 3, 2020
13:30 — 15:30

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Problem 4

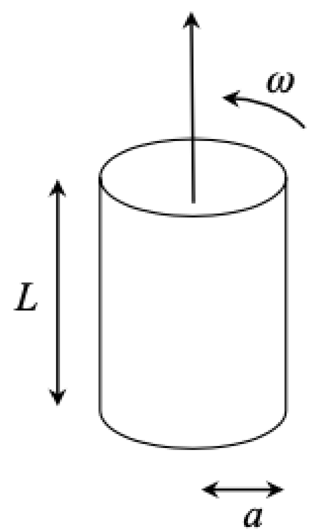
A cylindrical thin shell of electric charge has a length L and radius a, where $L \gg a$. The surface charge density on the shell is σ. The shell rotates about its axis with an angular velocity ω that increases with time $t > 0$ as $\omega = kt$, where k is a positive constant. You may neglect fringe and radiation effects.

(a) Find the direction and magnitude of the magnetic field $\vec{B}$ everywhere inside the cylinder for $t > 0$. [4 points]

(b) Find the direction and magnitude the electric field $\vec{E}$ inside the cylinder for $t > 0$. [2 points]

(c) State the general formula for the Poynting vector $\vec{S}$ in terms of $\vec{E}$ and $\vec{B}$. What does $\vec{S}$ mean? [2 points]

(d) Find the direction and magnitude of the Poynting vector for this system. [2 points]



Problem 5

A point charge Q is placed at the point (a, a, a) in the positive octant and the positive portions of the coordinate planes are made up by three mutually orthogonal infinite plane conductors which are maintained at zero potential each.

(a) How many image charges are needed to satisfy the boundary conditions? [1 point]

(b) What are the locations of these image charges in terms of their Cartesian coordinates (x, y, z)? [2 points]

(c) Determine the magnitude of the force on the charge Q. [4 points]

(d) Determine a unit vector in the direction of the force on the charge Q. [3 points]

Problem 6

Consider a cylindrical capacitor with length L. Dielectric materials with a permittivity ϵ are filled between an inner core wire with a radius a and an outer conducting shell with a radius b.

(a) When the capacitor is charged with electric charge Q, compute the electric field as a function of the radius r between the wire and conducting shell. [3 points]

(b) Compute the electric capacity of this capacitor. [4 points]

(c) If this capacitor is connected with an external battery with voltage V and we pull out some part of the dielectric material from the capacitor, with a displacement x, compute the force needed to keep the dielectric material still. Which direction is this force pointing to? [3 points]

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PART III

Monday, January 6, 2020
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

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Problem 7

A cylinder, with thermally insulated walls, contains a movable frictionless insulated piston that divides the cylinder into two compartments. Each compartment contains one mole of an ideal monatomic gas. The initial pressure P_i , initial volume V_i , and initial temperature T_i are the same on both sides of the piston. Now, using a heating coil, located inside the left compartment, heat is slowly supplied until the pressure in the right compartment increases to a final value of $32P_i$. In terms of gas constant R and T_i,

(a) What are the final temperature T_{fR} and final volume V_{fR} of the right compartment? [2 points]

(b) What are the final temperature T_{fL} and final volume V_{fL} of the left compartment? [2 points]

(c) How much work is done on the gas in the right compartment? [2 points]

(d) How much heat is supplied to the gas in the left compartment? [2 points]

(e) How much does the entropy change in each compartment? [2 points]

USEFUL INFORMATION: For an ideal monatomic gas $C_v = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$.

Problem 8

A photon of energy E collides with a proton at rest to produce a neutron and a charged pion by the reaction

$$\gamma + p \longrightarrow n + \pi^+$$

(a) Find the minimum energy of the photon for pion production. Assume the proton and neutron have roughly the same mass M while the pion has mass m. [7 points]

(b) Find the velocity of the center of momentum of the system in the lab frame. [3 points]

Problem 9

A system is composed by N independent particles. There are only two energy levels for the particles, $e_1 = 0$ and $e_2 = e > 0$. There are N_1 particles in the e_1 state and $N - N_1$ in the e_2 state. The total energy of the system is $E = (N - N_1)e$.

(a) Compute the system's entropy. You may use the Stirling approximation, $\ln x! = x(\ln x - 1)$. [4 points]

(b) Derive the system's temperature as a function of its energy. Under what circumstance, the system's temperature is negative, $T < 0\ K$? [4 points]

(c) When the negative temperature system contacts with a system with positive temperature, what direction does the heat flow? Why? [2 points]

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PART IV

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Problem 10

The quantum mechanical state of a system at $t = 0$ is a linear combination of the orthonormalized ground state and the first excited state of a one-dimensional harmonic oscillator (of mass M and frequency ω), that is,

$$\Psi(x,0) = N(\psi_0 + \psi_1)$$

(a) Determine the constant N so that the state $\Psi(x,0)$ is normalized at $t = 0$. [1 point]
(b) What is the state function $\Psi(x,t)$ at a later time $t > 0$. [3 points]
(c) Evaluate the expectation values of the position, $\langle x \rangle_t$, and the momentum, $\langle p \rangle_t$, of the system at any time t. [4 points]
(d) Show that,

$$\frac{d\langle x \rangle_t}{dt} = \frac{\langle p \rangle_t}{M}.$$

[2 points]

USEFUL INFORMATION: If ψ_n is the eigenfunction of a one-dimensional linear harmonic oscillator corresponding to eigenvalue $E_n = (n + \frac{1}{2})\hbar\omega$, then

$$\int_{-\infty}^{+\infty} \psi_m^* x \psi_n dx = \sqrt{\frac{\hbar}{2M\omega}} \left[\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}\right]$$

and

$$\int_{-\infty}^{+\infty} \psi_m^* p \psi_n dx = \sqrt{\frac{\hbar M\omega}{2}} i \left[\sqrt{n+1}\,\delta_{m,n+1} - \sqrt{n}\,\delta_{m,n-1}\right]$$

Problem 11

The wave function of a one-dimensional quantum system is

$$\psi(x) = \frac{N}{x^2 + a^2}$$

with $-\infty \leq x \leq +\infty$.

(a) What value of N normalizes the wave function $\psi(x)$? [1 point]

(b) Using normalized $\psi(x)$, determine the expectation values $\langle x \rangle$, $\langle x \rangle^2$ as well as Δx. [4 points]

(c) Using normalized $\psi(x)$, also determine the expectation values $\langle p \rangle$, $\langle p \rangle^2$ as well as Δp. [4 points]

(d) Explicitly verify that the uncertainty principle, $\Delta x \Delta p \geq \frac{\hbar}{2}$, is satisfied. [1 point]

USEFUL INFORMATION: Consider the integral

$$I_n = \int_0^\infty \frac{dx}{(x^2 + a^2)^n}.$$

For $n = 1$,

$$I_1 = \frac{\pi}{2a}.$$

For $n \geq 2$,

$$I_n = \frac{2n-3}{2(n-1)} \frac{1}{a^2} I_{n-1}.$$

Problem 12

Consider a particle with energy $E = \frac{\hbar^2 k^2}{2m}$ going through a potential barrier which is composed of two δ-functions, $V(x) = V_0[\delta(x) + \delta(x-a)]$ $(V_0 > 0)$.

(a) Write down the Schrödinger equation for the particle's wave function and derive the general form of the solution. [4 points]

(b) Derive the boundary conditions at $x = 0$ and $x = a$. [3 points]

(c) Using the boundary conditions, compute the reflection and transmission coefficients. [3 points]