Ph.D. QUALIFYING EXAMINATION
DEPARTMENT OF PHYSICS AND ASTRONOMY
WAYNE STATE UNIVERSITY

PART I

Friday, May 5, 2023
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

INSTRUCTIONS: This examination consists of three problems each worth 10 points. Use a separate booklet for each problem. Write the following information on the front cover of each booklet:

1. your special ID number that you received from Delores Cowen,
2. the problem number (*i.e.* Problem 7).

Please make sure your answers are dark and legible.

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Problem 1 (10 points)

A railroad car can move on a frictionless track. The railroad car has mass M and is initially at rest. In addition, N people (each mass m) are initially standing at rest on the car.

a) Consider the case where all N people run to the end of the railroad car in unison and reach a speed, relative to the car, of V_r. At that point they all jump off at once. Calculate the velocity of the car relative to the ground, after all the people have jumped off. [3 Points]

b) Now consider a different case, in which people jump off one at a time with relative speed V_r, while the remaining people remain at rest relative to the car. That continues until all N people have jumped off. Find an expression for the final velocity of the railroad car relative to the ground. [6 Points]

c) In which case does the railroad car attain a greater velocity? [1 Point]

Problem 2 (10 points)

Consider circular orbits in the field of a central force $f(r) = -kr^n$

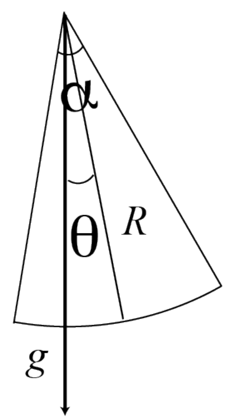
a) For circular orbits, what is the relation between orbital velocity v and radius r? [3 Points]

b) For what values of n are these circular orbits stable to small radial perturbations? [5 Points]

c) Specially, are the circular orbits stable or unstable for n=-3? [2 Points]

Problem 3 (10 points)

A section of a flat uniform disk of radius R, apex angle α, and mass m is suspended from the top as a compound pendulum under the influence of gravity g. We denote by θ the angle of displacement from the vertical. We assume that this system is released from rest at an initial displacement angle of θ_0.
a) Find the moment of inertia of this compound pendulum. [2 Points]
b) Find its kinetic energy in terms of $\dot{\theta}$. [2 Points]
c) Find its potential energy as function of θ. [2 Points]
d) Using the Lagrangian formalism, and the results of parts (b) and (c), find the equation of motion for θ. Calculate the angular frequency ω_0 in the limit of small oscillations. [4 Points]



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PART II

Friday, May 5, 2023
13:30 — 15:30

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Problem 4 (10 points)

An electromagnetic wave moves in vacuum with frequency ω. The electric field is polarized along the $z-$axis with maximum amplitude E_0. At $\mathbf{r} = t = 0$ the electric field has magnitude E_0, while the magnetic field has the form $\mathbf{B} = (0, a, b)$, where a and b are zero or positive quantities.The direction of motion is defined by $\mathbf{k}$.

a) Write an expression for $\mathbf{E}(\mathbf{r}, t)$ over all space and time, as a function of $\omega, E_0, \mathbf{k}$ and c (3pts)

b) Find the magnetic field components and $\mathbf{B}(\mathbf{r}, t)$ over all space and time, as a function of $\omega, E_0, \mathbf{k}$ and c.(3pts)

c) Find $\mathbf{k}$ and state the direction of propagation of the wave. (4pts)

Problem 5 (10 points)

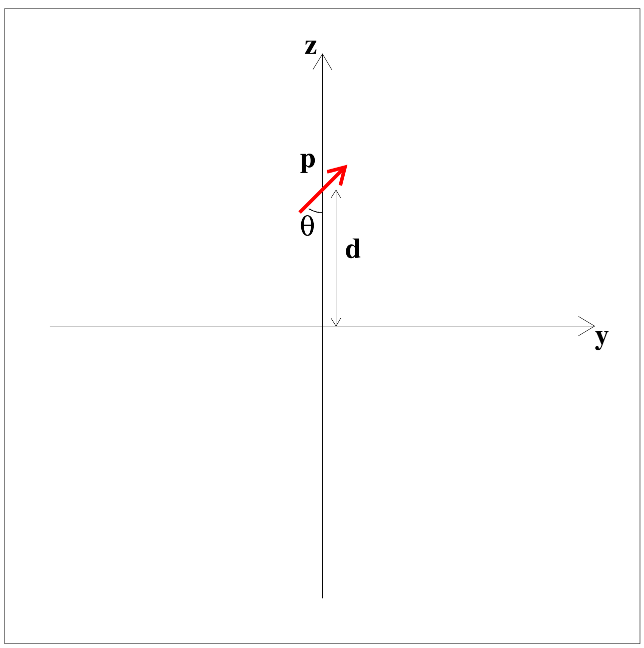
An electric dipole of magnitude p is located a distance d from and infinite grounded conducting plane which is in the (x, y) plane. The dipole makes an angle θ with the perpendicular to the plane. See Figure.

a) compute the electric potential everywhere. (2pts.)

b) given the result of a), compute the potential at the plane surface. (3pts)

c) derive an expression for the torque acting on the dipole. (3 pts).

d) for which values of θ is the dipole in a stable equilibrium? (2 pts.)

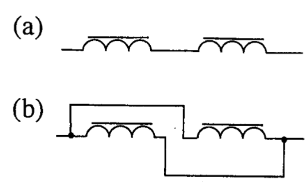


Problem 6 (10 points)

Two solenoids are connected in series, as shown in part a) of the Figure, and connected to a power source. The individual inductances are 1 and 2 mH, but the inductance of the entire circuit in part a) is 3.6 mH.

a) Calculate the mutual inductance in Fig. a). (4 pts.)

b) Without changing the positions (and therefore the mutual inductance), the solenoids are now connected in parallel, part b) of the figure. Calculate the total inductance of the circuit in Fig. b). (6 pts.)



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PART III

Monday, May 8, 2023
10:00 — 12:00

ROOM 245 PHYSICS RESEARCH BUILDING

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Problem 7 (10 points)

Electrons confined to the surface layer of a semiconductor can move in only two-dimensions. Assume N such electrons occupy an area A and that they have spin $1/2$, mass m, and single-particle energy $\epsilon = p^2/2m$.

(a) (3 points) Find the Fermi energy ϵ_F and the density of states $\mathcal{D} = dN/d\epsilon$.

(b) (3 points) Find the total energy E at $T = 0$.

(c) (4 points) Write $\langle N \rangle$ for $T > 0$ and chemical potential μ as an integral over the Fermi distribution. Find $\mu(T)$ by doing the integral and setting it equal to $\langle N \rangle = N$. Show that $\mu \to \epsilon_F$ as $T \to 0$.

Problem 8 (10 points)

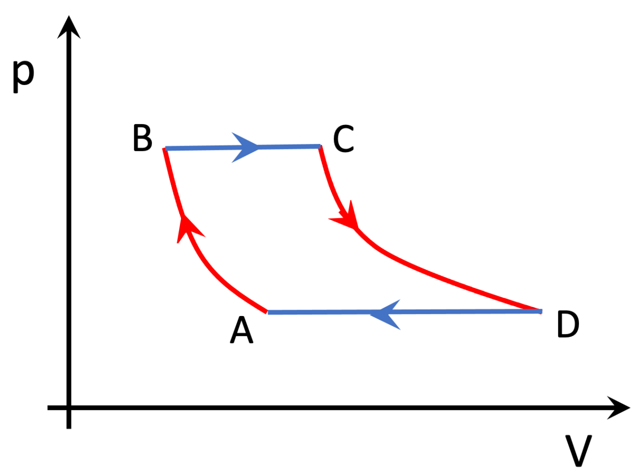
Gas turbine engines like those in jet aircraft follow the Brayton cycle, see the figure. AB and CD are adiabatic compression and expansion, respectively. BC and DA are constant pressure processes. Assume that the working fluid is an ideal gas with constant ratio $\gamma = c_p/c_v$. Note that $pV^\gamma =$ constant in an adiabatic process.

(a) (1 point) How is the efficiency of the engine defined? Use the first law of thermodynamics to write the efficiency in terms of the heat flow into the gas Q_H and the exhaust heat Q_L.

(b) (4 points) Show that the efficiency is

$$\epsilon = 1 - \left(\frac{T_D - T_A}{T_C - T_B}\right)$$

(c) (5 points) Find the efficiency in terms of γ and the pressure ratio $r_p = p_B/p_A$.



Problem 9 (10 points)

A photon of frequency ω is emitted in the x direction by a source that is at rest in the frame S. Suppose that the photon is detected by an observer in a frame S' that moves relative to S with velocity $v = v\hat{x}$.

(a) What is the energy and momentum of the photon in terms of ω, Planck's constant $\hbar$ and the speed of light c in the frame S(2 points)?

(b) Find the energy and momentum in the frame S'(2 points).

(c) Find the frequency ω_1 of the photon as measured by the observer in a frame S'. The shift of ω_1 relative to ω is called the "longitudinal" Doppler effect because the photon moves in the same direction as the observer(3 points).

(d) Suppose that a second photon now propagates in the y direction. Find the frequency ω_2 of this photon in S', which still moves relative to S with $v = v\hat{x}$. This is called the "transverse" Doppler effect (2 point).

(e) Show that the longitudinal shift from part c reduces to the classical Doppler formula by expanding in powers of v/c and dropping terms of order v^2/c^2 and higher. Explain why the transverse Doppler effect is a purely relativistic effect (1 point).

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PART IV

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Problem 10 (10 points)

Consider a particle of mass m in the potential $U(x) = -\alpha\delta(x) + U_0\,\theta(x)$. Assume $\alpha > 0$ and $U_0 > 0$.; Heaviside step function $\theta(x) = 1$ for $x > 0$ and zero otherwise.

(a) Find the general solution of the Schrodinger equation for energies corresponding to bound and unbound states (3 pts).

(b) Determine the condition for the existence of a bound state (3 pts)

(c) Find the energy of the bound state(s)(4 pts).

Problem 11 (10 points)

Consider a charged particle in the ground state of the oscillator potential $U = m\omega^2/2$ under the influence of a weak electric field pointing along the direction of oscillation.

(a) Treating the interaction of the charge with electric field as perturbation, find the corrections to the energy level in the first order (3 pts)

(b) Find the second order correction to the energy level(4 pts).

(c) Compare with the exact solution (3 pts).

You might find the following equations useful:

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x) \tag{1}$$

$$\begin{aligned} \hat{a}_{+}|n\rangle &= \sqrt{(n+1)}\,|n+1\rangle; \\ \hat{a}_{-}|n\rangle &= \sqrt{n}\,|n-1\rangle \end{aligned} \tag{2}$$

Problem 12 (10 points)

The electron in a hydrogen atom occupies the combined spin and position state described by the wave function

$$R_{21}\left(\sqrt{\frac{1}{3}}Y_{1,0}\chi_{-}+\sqrt{\frac{2}{3}}Y_{1,-1}\chi_{+}\right), \tag{3}$$

where $Y_{l,m}$ are spherical harmonics, and $\chi_{\pm}$ are the spin wave functions corresponding to $s_z = \pm 1/2$.

(a) If one measured the orbital angular momentum squared, L^2, what values might one get, and what is the probability of each(1 pts)?

(b) Same for the "z"-component of the orbital momentum(1 pts).

(c) Same for the spin angular momentum squared(1 pts).

(d) Same for the z-component of the spin angular momentum(1 pts).

(e) Same for the total angular momentum squared(3 pts).

(f) Same for the z-component of the total angular momentum(3 pts).

You might find the following equation useful:

$$\hat{l}_{\pm}|l,m\rangle = \sqrt{l(l+1)-m(m\pm 1)}\;|l,m\pm 1\rangle \tag{4}$$