MAT 1050 GROUP FINAL EXAM – WINTER 2006

SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:

\[
\frac{1}{5^x} + 2 \cdot 3^x - 7 \sqrt{x} - \frac{1}{6^x} + 2 \sqrt{x} + 3 \cdot 5^x
\]

2. (7 pts.) Simplify completely:

\[
\left( \frac{2x^{-4}y^2}{z^{-3}} \right)^{-2} \left( \frac{8y^4z^{-3}}{5x^{-2}} \right)
\]

3. (7 pts.) Simplify completely:

\[
\sqrt[3]{9} \left( \sqrt[3]{18} + \sqrt[3]{6} \right) - \sqrt[3]{4} \left( \sqrt[3]{4} + \sqrt[3]{12} \right)
\]

4. (7 pts.) Simplify completely:

\[
\left( \sqrt[4]{49^{-1}} - \sqrt[4]{16^{-1}} \right)^{-1}
\]

5. (6 pts.) Solve:

\[-3 \left| \frac{3x-1}{3} \right| - 2 = 7
\]

6. (6 pts.) Solve:

\[-4 \left| \frac{3x+1}{5} \right| \geq -12
\]

7. (7 pts.) Ashley invested $10,000 for one year, part at 7% interest and part at 5% interest. If she earned a total interest of $540, how much was invested at each rate?

8. (7 pts.) Solve for \( r \):

\[
\frac{1}{p} = \frac{pr-q}{pr} + q
\]

9. (6 pts.) Let \( f \) be the function given by \( f(x) = \frac{2 \cdot 4 \sqrt{3x-1}}{5} \).

What is the domain of \( f \)?

10. (6 pts.) Let \( f \) be the function given by \( f(x) = \frac{\sqrt{2x^2 + 1} - 4}{x} \).

Find and simplify \( f(2) - f(-2) \).
11. (7 pts.) Let \( f \) be the function given by \( f(x) = x^2 - x - 1 \).

Find and simplify \( \frac{f(x - 2h) - f(x - 4h)}{2h} \).

12. (6 pts.) Find the equation of the line that is perpendicular to the line \( y = 4 \) and passes through the point \( \left( \frac{1}{2}, \frac{3}{2} \right) \).

13. (6 pts.) Find the equation of the line that is parallel to the line \( 2x + 5y = 10 \) and passes through the point \( (5, -7) \).

14. (7 pts.) A rectangular garden has a width of 8 feet. The path that leads diagonally across the garden is 12 feet. Find the area of the garden.

15. (7 pts.) Solve, writing any non-real solutions in the form \( a + bi : x^3(x + 4) = -21x \)

16. (7 pts.) Graph, labeling the vertex and all \( x \) and \( y \) intercepts: \( f(x) = 2x^2 - 4x + 4 \)

17. (7 pts.) Simplify completely:

\[
\frac{-1}{x+1} + \frac{x-1}{1} = \frac{-1 + x - 1}{x+1} = \frac{-2}{x+1} = \frac{x^2 - 2x - 3}{4}
\]

\[
\frac{1}{x-1} - \frac{4}{x^2 - 4x + 3}
\]

18. (7 pts.) Solve: \( \sqrt{3 - 2x} + \sqrt{3x + 13} = 5 \)

19. (7 pts.) Solve: \( (x + 1)^2 \leq 0 \)

20. (7 pts.) Solve: \( \frac{2}{x+3} < \frac{2}{x-1} \)

21. (7 pts.) The graph of a function, \( f \), is shown here.

a) What is the domain of \( f \)?

b) What is the range of \( f \)?

c) For what values of \( x \) does \( f(x) = -1 \)?
22. (7 pts.) Solve: \[2x^{-\frac{2}{3}} - x^{-\frac{1}{3}} - 3 = 0\]

23. (6 pts.) Find: a) \(\log_5\left(\frac{1}{125}\right)\) b) \(\log_{10}(0.01)\) c) \(\log_{16}\left(\frac{1}{4}\right)\)

24. (6 pts.) Using the approximate values \(\log_4(3) = 0.8\) and \(\log_4(15) = 1.9\) find:
   a) \(\log_4\left(\frac{1}{5}\right)\) b) \(\log_4(27)\) c) \(\log_4(12)\)

25. (7 pts.) Let \(f(x) = \log_5(2x + 1) - \log_5(x^2 - 2)\). Find all \(x\)-intercepts of \(f\).

26. (7 pts.) Identify and sketch the curve given by \(2(x - 1)^2 + 18(y + 3)^2 = 18\).

27. (7 pts.) Arrange the following numbers in order from smallest to largest:
   \[\tan\left(\frac{\pi}{4}\right), -\frac{\pi}{3}, \sin(3.2), \cos(3.2)\]

28. (6 pts.) a) Convert to radians: \(-72^\circ\)
   b) Convert to degrees: \(\frac{5}{3}\) radians

29. (6 pts.) For the right triangle shown here, find \(x\).

30. (7 pts.) A car and a motorcycle leave the rest area at the same time, with the motorcycle traveling 10 mph slower than the car. The car travels 130 miles in the same time it takes the motorcycle to travel 110 miles. Find the speed of each vehicle.