MAT 1050 GROUP FINAL EXAM – WINTER 2007

SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:

\[ 3^x + \sqrt{3} \cdot x - 2 \cdot 3^x + 6^x + \sqrt{2} \cdot x - 5 \cdot 3^x \]

2. (7 pts.) Simplify completely:

\[ \frac{\sqrt[3]{54} a^5 b^9 \cdot \sqrt[3]{a^4} c^{-4}}{\sqrt[3]{2} b^{-9} c^2} \]

3. (7 pts.) Simplify completely:

\[ (2 \cdot \sqrt[3]{2} + \sqrt[3]{3}) \left( \frac{\sqrt[4]{4} - 2 \cdot \sqrt[6]{5}}{3} \right) \]

4. (7 pts.) Simplify completely:

\[ \left( \frac{32 \cdot \frac{3}{5} + 8 \cdot \frac{2}{3}}{8 \cdot \frac{2}{3}} \right)^{-1} \]

5. (6 pts.) Solve: \[ |3 + x| = |x - 4| \]

6. (6 pts.) Solve: \[ 4 - |3a + 1| < 1 \]

7. (7 pts.) In a political science classroom, the number of Republicans is \( \frac{2}{3} \) the number of Democrats, and the number of Independents is \( \frac{1}{4} \) the number of Republicans. If there are 66 students in the class, how many are Independents?

8. (7 pts.) Solve for \( A \): \[ \frac{A + B}{A} = \frac{1}{AB} + 2 \]

9. (6 pts.) Let \( f \) be the function given by \( f(x) = \frac{x^3}{2} + \frac{\sqrt{3} - x}{4} \).

What is the domain of \( f \)?

10. (6 pts.) Let \( g \) be the function given by \( g(x) = \frac{x + 5}{x - 1} \).

Find and simplify \( 3g(2) - [g(0)]^2 \).
11. (7 pts.) Let \( h \) be the function given by \( h(x) = x^2 + 2x \).

Find and simplify \( \frac{h(x + a) - h(x)}{a} \).

12. (6 pts.) Find the equation of the line with \( x \)-intercept \((5,0)\) and \( y \)-intercept \((0, -3)\).

13. (6 pts.) Find the equation of the line that is perpendicular to the line \( 4y - 3x = 5 \) and passes through the point \((2, -3)\).

14. (7 pts.) Three consecutive odd integers are such that the sum of the squares of the first two integers is 54 more than twenty times the third integer. Determine the three integers.

15. (7 pts.) Solve, writing any non-real solutions in the form \( a + bi \): \( (2x^2 + 10)(x^2 + 4x + 2) = 0 \)

16. (7 pts.) Graph, labeling the vertex and all \( x \) and \( y \) intercepts: \( k(x) = -3x^2 + 12x \)

17. (7 pts.) Simplify completely: \( \frac{x}{x + 2} + \frac{1}{3x + 2} + \frac{1}{x} \)

18. (7 pts.) Let \( f(x) = \sqrt{3x - 3} - \sqrt{x} \). Find all \( a \) such that \( f(a) = 1 \).

19. (7 pts.) Solve: \( 3x > 5 - 2x^2 \)

20. (7 pts.) Solve: \( \frac{x^2}{x - 2} \geq 0 \)

21. (7 pts.) The graph of a function, \( f \), is shown here.

a) What is the domain of \( f \)?

b) What is \( f(1) \)?

c) For what values of \( x \) does \( f(x) = 2 \)?
22. (7 pts.) Solve: \( |x + 1|^2 + |x + 1| - 6 = 0 \)

23. (6 pts.) Find: 
   a) \( \log_{32} (2) \) 
   b) \( \log_{\sqrt{11}} (1) \) 
   c) \( \log_{\frac{1}{3}} (27) \)

24. (6 pts.) Using the approximate values \( \log_7 (4) = 0.7 \) and \( \log_7 (20) = 1.5 \) find:
   a) \( \log_7 \left( \frac{4}{7} \right) \) 
   b) \( \log_7 (16) \) 
   c) \( \log_7 (25) \)

25. (7 pts.) Solve: \( \log_2 (x - 1) = 2 - \log_2 (3x - 2) \)

26. (7 pts.) Identify and sketch the curve given by \( 3y^2 + 27x^2 = 27 \).

27. (7 pts.) Arrange the following numbers in order from smallest to largest:
   \( \sin(0.2) \quad \cos(0.2) \quad \sin \left( \frac{-3\pi}{2} \right) \quad \frac{-3\pi}{2} \)

28. (6 pts.) Convert to degrees:
   a. \( \frac{5\pi}{6} \) radians
   b. \( \frac{4}{3} \) radians

29. (6 pts.) For the right triangle shown here, find an exact value for:
   a. \( \sin(\angle A) \)
   b. \( \cos(\angle A) \)
   c. \( \tan(\angle B) \)

30. (7 pts.) Lamont walks the 15 meters from the front door of his house to his car at a constant rate of speed. Having forgotten to lock the front door, Lamont runs the 15 meters back to the front door along the same path. His speed running is 2 m/sec faster than his speed walking. If the total time to walk to the car and run back to the door was 8 seconds, what was Lamont’s walking speed?