MAT 1800 Final Exam
SHOW ALL WORK IN A BLUEBOOK:
Only minimal credit will be awarded for answers without supporting work.

#1 (10 pts.) Solve: \[ \frac{3x}{x-1} < \frac{x}{x+4} + 3 \]

#2 (8 pts.) Find the domain of the function: \[ f(x) = \frac{\sqrt{2x^2 - x}}{4 - x^2} \]

#3 (10 pts.) Given that \( f(x) = \frac{1}{x+1} \) and \( g(x) = x - 1 \) simplify the following completely:

a) \[ \frac{f(x+h) - f(x)}{h} \]

b) \( (f \circ g)(x) \)

#4 (10 pts.) An open top box with a square base (see figure) is to be constructed from 84 square inches of plywood. The height of the box is 2 inches. What are the dimensions of the box?

#5 (10 pts.) Graph \( f(x) = x^3 - 4x \). Then find the average rate of change of this function from \( x = 4 \) to \( x = -3 \).

#6 (12 pts.) The number \( N \) of bacteria in an antibiotic treated culture is modeled by the function \( N(t) = 140e^{kt} \), where \( t \) is measured in hours. If there are 20 bacteria left after 3 hours, find the exact time required for the there to be only 7 bacteria left.
#7 (12 pts.) Given that 3 and 2i are roots of the equation \( x^5 - 4x^4 + x^3 + 2x^2 - 12x + 72 = 0 \) find its complete solution set.

#8 (12 pts.) Graph \( f(x) = \frac{4x^2 - 12x}{x^2 - 25} \) labeling all intercepts and asymptotes.

#9 (10 pts.) Graph \( f(x) = 2\ln(1 - 3x) - 1 \) labeling all intercepts and asymptotes.

#10 (10 pts.) Given the approximate values: \( \log_{10}(2) = 0.3 \) and \( \log_{10}(3) = 0.47 \)

Find a) \( \log_{10}(5) \) b) \( \log_{10}(12) \)

#11 (12 pts.) Solve: \( 2\log_8(x) - \log_8(3x + 8) - \log_8(x - 4) + \frac{1}{3} = 0 \)

#12 (8 pts.) Find the values: a) \( \sec\left(\frac{19\pi}{6}\right) \) b) \( \csc\left(-\frac{16\pi}{3}\right) \)

#13 (12 pts.) Graph \( f(x) = 4\sin\left(2x + \frac{\pi}{3}\right) + 1 \) labeling the highest and lowest points.

#14 (10 pts.) Given that \( \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} \) and \( \sin \theta = \frac{1}{5} \) find the value of: \( \sin\left(\theta - \frac{\pi}{3}\right) \)

#15 (12 pts.) Solve for \( x \): \( 2\cos^2 x - \tan x \sin x = 1 - \sec x \) where \( 0 \leq x < 2\pi \).

#16 (10 pts.) Find the values: a) \( \tan\left(\sin^{-1}\left(-\frac{2}{3}\right)\right) \) b) \( \cos^{-1}\left(\cos\left(\frac{9\pi}{8}\right)\right) \)

#17 (10 pts.) Prove the identity: \( (\tan \theta - \sec \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta} \)
#18 (8 pts.) Simplify:  

a) \(27^{-2 \log_3(2)}\)  

b) \(3e^{\ln(10)} + 2\ln(e^{10}) - e^{3\ln(10)}\)

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#19 (8 pts.)

A rectangle is inscribed in a semicircle of radius 10 inches (see figure). Find a function that models the area \(A\) of the rectangle in terms of \(t\) and \(\theta\).

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#20 (6 pts.) Given the point \((-3, \sqrt{3})\) in rectangular coordinates, convert it to polar coordinates \((r, \theta)\) where \(r > 0\) and \(0 \leq \theta \leq 2\pi\).