MAT 1050 GROUP FINAL EXAM – FALL 2008

SHOW ALL WORK. DO NOT USE A CALCULATOR.

1. (7 pts.) Simplify by adding (or subtracting) like terms wherever possible:

\[ 3y^2 + \sqrt{11}y^2 - 2y^2 + y^2 + y^2 - y^2 \]

2. (7 pts.) Simplify completely:

\[ (-3x^3yz)^2 \left( \frac{x^2z^{-3}}{4x^0y^{-2}z^2} \right) \]

3. (7 pts.) Multiply and simplify completely:

\[ \left( 3\sqrt{3} - \sqrt{5} \right) \left( 2\sqrt{3} + 3\sqrt{5} \right) \]

4. (7 pts.) Simplify completely:

\[ \left( \frac{25^{-\frac{1}{2}} + 49^{-\frac{1}{2}}}{25^{-\frac{1}{2}} - 49^{-\frac{1}{2}}} \right)^2 \]

5. (6 pts.) Let \( f(x) = |4 - x| + 3 \). Find all inputs, \( x \), such that \( f(x) < 2 \).

6. (6 pts.) Solve:

\[ \frac{-3|2x + 5|}{7} \leq -3 \]

7. (7 pts.) At 1:00 PM, a train leaves a station travelling due east at a constant speed. One hour later, a second train leaves the same station, travelling due west, at a speed that is 16 mph more than the first train. At 5:00 PM, the two trains are 384 mile apart. Find the speed of each train.

8. (7 pts.) Solve for \( a \):

\[ \frac{a - 2x}{ax} - 1 = \frac{2}{a} \]

9. (6 pts.) Let \( f \) be the function given by \( f(x) = \frac{\sqrt{2} - 2x}{\sqrt{5}} \).

What is the domain of \( f \)?

10. (6 pts.) Let \( g \) be the function given by \( g(x) = \frac{20 - x^2}{\sqrt{x} + 4} \).

Find and simplify \( [g(0) + 3g(5)]^2 \).
11. (7 pts.) Let \( f \) be the function given by \( f(x) = -x^2 + 2x + 4 \).

Find and simplify \( \frac{f(a + 2h) - f(a)}{2h} \).

12. (6 pts.) Find the equation of the line that is perpendicular to the line \( 4x - 3y = 6 \) and passes through the point \((-8,0)\).

13. (6 pts.) Find the equation of the line containing the points \((-3,1/2)\) and \((-3,5/2)\).

14. (7 pts.) In a right triangle, the hypotenuse is 4 inches less than 3 times the shorter leg. The longer leg is 4 inches longer than twice the shorter leg. Find the perimeter of the triangle.

15. (7 pts.) Solve, writing any non-real solutions in the form \( a + bi \): \( x^2(x^2 + 6) = -4x^3 \)

16. (7 pts.) Graph, labeling the vertex and all \( x \) and \( y \) intercepts: \( f(x) = -2x^2 + 6 \)

17. (7 pts.) Simplify completely: \( \frac{x + (x + 2)^{-1}}{1 - (x + 2)^{-1}} \)

18. (7 pts.) Solve: \( \sqrt{6x - 18} = 1 + \sqrt{2x - 2} \)

19. (7 pts.) Solve: \( (2x + 1)(x + 1) \leq 6 \)

20. (7 pts.) Solve: \( \frac{1}{3 - x} \leq \frac{-1}{4 + x} \)

21. (7 pts.) The graph of a function, \( f \), is shown here.

   a) What is the domain of \( f \)?

   b) What is the range of \( f \)?

   c) What is \( f(2) \)?

   d) Find all \( x \) such that \( f(x) = -3 \).
22. (7 pts.) Solve: \(2x^3 + 20 = 14x^3\)

23. (6 pts.) Find: a) \(\log_{27}(3)\)  
   b) \(\log_{\frac{1}{5}}(125)\)  
   c) \(\log_b\frac{1}{b}\)

24. (6 pts.) Using the approximate values \(\log_{10}(2) = 0.3\) and \(\log_{10}(14) = 1.1\) find:
   a) \(\log_{10}(140)\)  
   b) \(\log_{10}\frac{1}{7}\)  
   c) \(\log_{10}(16)\)

25. (7 pts.) Let \(f\) be the function given by \(f(x) = \log_b(x^2 + 1) - 2\log_b(x)\).

   Find all \(x\) such that \(f(x) = 1\).

26. (7 pts.) Identify and sketch the curve given by \(x^2 - 6x + y^2 + 8y = -16\).

27. (7 pts.) Arrange the following numbers in order from smallest to largest:
   \(\sin(1.5)\) \(\tan(\pi)\) \(\cos(1.5)\) \(\cos(-\pi)\)

28. (6 pts.) a. Convert to radians: \(10\pi^o\)
   
   b. Convert to degrees: \(\frac{1}{3}\) radians

29. (6 pts.) For the triangle shown here, find an exact value for \(x\):

   ![Triangle Diagram]

30. (7 pts.) The distance between Port A and Port B is 40 miles along a river with a current of 5 mph.

   If a boat goes upstream from Port A to Port B and back in a total of 6 hours, what is the boat’s speed in still water?